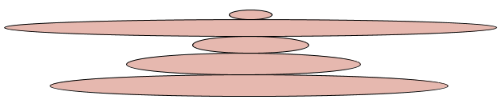
1 Pancake Flipping

[1]

Harry Dweighter is a waiter navigating a busy restaurant with a stack of pancakes, all of different sizes. To avoid disaster, Harry wants to sort the pancakes in order by size, with the biggest pancake on the bottom. Having only one free hand, the only available operation is to lift a top portion of the stack with a spatula, turn it upside down, and replace it. Harry wants to figure out how many times will he have to flip to get the pancakes in order, if he flips as efficiently as possible.

1. Find the minimum number of flips needed to reorder this stack of 5 pancakes.

   Harry Dweighter wants to make sure he can deliver the pancakes before they get cold, so he would like to know the worst-case-scenario number of flips he'd have to perform on any given number n of pancakes. We'll call this the pancake number $P(n)$.

   For example, for two pancakes, he might need 0 flips, if the pancakes happen to be in order already, but he might need 1 flip. He won't need any more than 1 flip. So for two pancakes, the pancake number $P(2) = 1$.

2. For three pancakes, what are all the possible ways of arranging the pancakes? For each way, find the minimum number of flips needed to order this stack.

3. What is $P(3)$?

4. What is $P(4)$ and which order(s) of four pancakes take the most flips to put in order?

5. Try to find an algorithm (a systematic method, or set of instructions for the waiter) that will always get any stack of pancakes in order. How many flips will your algorithm take, worst case, for a stack of 4 pancakes? 5 pancakes? 6 pancakes? 7 pancakes? n pancakes?

6. Find an upper bound on $P(n)$: the number of flips you need, worst case, for n pancakes. Your answer should be in terms of n.

7. Find a lower bound on $P(n)$. Remember, $P(n)$ is the number of flips needed to efficiently order a stack of n pancakes, in the worst case scenario for how the pancakes are originally ordered. To find a lower bound. try to find a tricky stack of pancakes for each size n that will take a lot of flips. Again, your answer should be in terms of n.

8. If the pancakes are burnt on one side, then the stack that gets sent out to the customer should not only be in order, but it should also have all the burnt sides facing down. What is the worst-case number of flips $B(n)$ for two, three, and four burnt pancakes?

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9. Can you find an upper bound on the burnt pancake number $B(n)$?
10. Can you find a lower bound on $B(n)$?