Polygonal Numbers [1]

Warm Up

1. Last time, we talked about trapezoidal numbers and some of their properties. Write down (or draw!) what it means for a number to be trapezoidal:

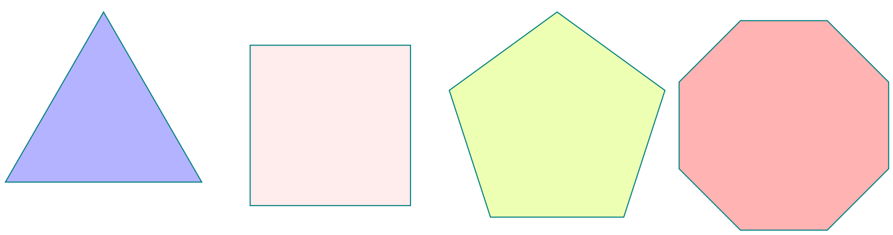
2. We also looked last time at how to compute the sum

$$5 + 6 + \ldots + 100,$$

and others like it. Is there is a geometric way we can find this sum?

Group Problems

From the warm up we see that trapezoidal numbers have a lovely geometric-algebra connection. Maybe other shapes have nice properties too. So, instead of trapezoids, maybe we want to look at squares, or pentagons. The first thing we have to do is *define* what it means for a number to be a square or a pentagonal number. Here are some example polygons to test your ideas.



1. For a number n, try come up with at least 3 definitions for what it means for a number to be an n-gon number. In your group, pick the one you think is best.
2. What sort of numbers are n-gonal?
3. Are there any n-gonal numbers that are also different polygonal numbers?
4. Earlier we saw that the area of a trapezoid gives us the arithmetic sum $a + (a + 1) + \ldots + (b - 1) + b$, were a and b are the two lengths of the trapezoid. Are there any nice patterns for this n-gonal number?

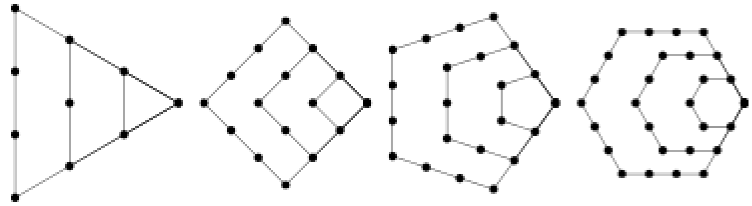


Image taken from mathworld.wolfram.com/PolygonalNumber.html

More Problems

1. Is every number some kind of polygonal number?
2. Is every number some kind of n-gonal number when $n \leq 10$?
3. Show that all hexagonal numbers are also triangular numbers.
4. So far we have only looked at *regular polygons*. How can we extend our definition even further to look at irregular polygons?
5. What other questions can we ask about these numbers? Write down at least 5 questions, and then try to answer at least one of them.

[1] Some problems are derived from Joshua Zucker's handout on Trapezoidal Numbers