

Logic Puzzles

Warm-up

On the Island of Knights and Knaves, there are two kinds of inhabitants: knights, who always tell the truth, and knaves, who always lie.

1. Would a person from the island of Knights and Knaves ever say “I am a Knave”?
2. There are 5 cards on a table.



Each card has a number on one side and a letter on the other. James states, “Any card with a vowel on one side has an even number on the other.” Tony only turned one card to show that James was wrong. Which card did Tony turn? Why does that work?

Colored hats, balls, and pencils

3. There are three boxes containing balls: the first one contains two white balls, the second – two black balls, and the third – a white ball and a black ball. The labels WW, BB, and WB were glued to the boxes so that none of the boxes has a correct label. Is it possible to choose one box so that after pulling one ball out of it, it is always possible to determine the contents of each box?¹

4. Three friends – sculptor White, violinist Black, and artist Red – met in a cafeteria. "It is remarkable that one of us has white hair, another one has black hair, and the third has red hair, though no one's name gives the color of their hair," said the black-haired person. "You are right," answered White. What color is the artist's hair?

5. In a box, there are pencils of at least two different colors, and of two different sizes. Prove that there are two pencils that differ both in color and in size.

6. Three people – A, B, and C – are sitting in a row in such a way that A sees B and C, B sees only C, and C sees nobody. They were shown 5 caps – 3 red and 2 white. They were blindfolded, and three caps were put on their heads. Then the blindfolds were taken away and each of the people was asked if they could determine the color of their caps. After A, and then B, answered negatively ("no, I can't tell what color my hat is", C replied affirmatively "yes, I know what color my hat is". How was that possible?

7. You are shown a set of four cards placed on a table, each of which has a number on one side and a letter on the other side. The visible faces of the cards show 3, 8, A and M. Which card(s) must you turn over in order to determine the truth of the statement: "If a card shows an even number on one face, then it has a vowel on the other side"?

¹These problems are from *Mathematical Circles: the Russian Experience*

And, Or, and Not

8. Your friend says she will bring apple slices and orange wedges to school the next day to share. Did she keep her promise if she:
 - Brought only apple slices?
 - Brought only orange wedges?
 - Brought only bananas?
 - Brought apple slices and orange wedges?
9. Your teacher says, "you must do some reading or do some math"
 - Do you need to do both?
 - Is it ok if you do both?
 - Is it ok if you do neither?
10. Your friend says, "I'm going to marry Harry or Joe."
 - Do you think she will marry both?
11. Your mom says, "You can have ice cream or cake for dessert."
 - Do you think she will let you have both?
12. Linda: I either have a dollar in my pocket or a frog in my hand.
Mary: That's not true!
If Mary is correct, then what do you know about what is in Linda's pocket and what is in her hand?
13. Linda: I have at least one flavor of ice cream in my freezer.
Mary: That's not true!
If Mary is correct again, then what do you know about Linda's freezer?
14. Linda: I have at least three kinds of vegetables in my refrigerator.
Mary: That's not true!
If Mary is correct again, then what do you know about Linda's freezer?

Knights and Knaves - Two residents ²

In the following problems, A and B are residents of the Island of Knights and Knaves.

15. A says: "B is a knave." B says: "A and I are knights. "
What are A and B?
16. A says: "Bob and I are not the same." B says: "Between Abe and me, exactly one is a knight."
What are A and B?
17. A makes the following statement: "At least one of us is a knave."
What are A and B?
18. Suppose A says, "Either I am a knave or B is a knight." What are A and B?
19. Suppose A says, "Either I am a knave or else two plus two equals five." What would you conclude?
20. Suppose A says, "I am a knave, but B isn't." What are A and B?
21. Once Mary visited the island of knights and knaves, and came across two of the inhabitants resting under a tree. She asked one of them, "Is either of you a knight?" He answered, and Mary knew the answer to her question.
What is the person to whom Mary addressed the question – is he a knight or a knave? And what is the other one? You have enough information to answer the question.
22. Suppose *you* visit the island of knights and knaves. You come across two of the inhabitants lazily lying in the sun. You ask one of them whether the other one is a knight, and you get a yes-or-no answer. Then you ask the second one whether the first one is a knight. You get a yes-or-no answer. Are the two answers necessarily the same?

²These problems are all taken from *What Is the Name of This Book* by Raymond Smullyan

Knights and Knaves, Three Residents

In the following problems, A, B, and C are all residents of the Island of Knights and Knaves.

23. Three of the inhabitants of the island of knights and knaves – A, B, and C – were standing together in a garden. A stranger passed by and asked A, "Are you a knight or a knave?" A answered, but rather indistinctly, so the stranger could not make out what he said. The stranger then asked B, "What did A say?" B replied, "A said that he is a knave." At this point the third man, C, said "Don't believe B; he is lying!"

The question is, what are B and C?

24. Suppose the stranger, instead of asking A what he is, asked A, "How many knights are among you?" Again, A answers indistinctly. So the stranger asks B, "What did A say?" B replies, "A said that there is one knight among us." Then C says, "Don't believe B; he is lying!"

Now what are B and C?

25. Again we have three people A, B, C, each of whom is either a knight or a knave. A and B make the following statements:

A: All of us are knaves.

B: Exactly one of us is a knight.

What are A, B, and C?

26. Suppose instead, A and B say the following:

A: All of us are knaves.

B: Exactly one of us is a knave.

Can it be determined what B is? Can it be determined what C is?

27. We again have three inhabitants, A, B, and C, each of whom is a knight or a knave. Two people are said to be of the *same type* if they are both knights or both knaves. A and B make the following statements:

A: B is a knave.

B: A and C are of the same type.

What is C?

28. Again three people A, B, and C. A says "B and C are of the same type." Someone then asks C, "Are A and B of the same type?" What does C answer?

Extra Problems: Knights, Knaves, and Normals

On another island, there are knights, knaves, and normals. The knights always tell the truth, the knaves always lie, and the normals sometimes tell the truth and sometimes lie.

29. We are given three people, A, B, C, one of whom is a knight, one a knave, and one normal (but not necessarily in that order). They make the following statements.

A: I am normal.
 B: That is true.
 C: I am not normal.

What are A, B, and C?

30. Two people, A and B, each of whom is either a knight, or knave, or a normal, make the following statements:

A: B is a knight.
 B: A is not a knight.

Prove that at least one of them is telling the truth, but is not a knight.

31. This time A and B say the following:

A: B is a knight.
 B: A is a knave.

Prove that either one of them is telling the truth but is not a knight, or one of them is lying but is not a knave.

32. On this island of knights, knaves, and normals, knaves are said to be of the *lowest* rank, normals of the *middle* rank, and knights of the *highest* rank.

Given two people A, B, each of whom is a knight, a knave, or a normal, they make the following statements.

A: I am of lower rank than B.
 B: That's not true!

Can the ranks of either A or B be determined? Can it be determined, of either of these statements, whether it is true or false?

33. Given three people A, B, C, one of whom is a knight, one a knave, and one normal. A, B make the following statements:

A: B is of higher rank than C. B: C is of higher rank than A.

Then C is asked: "Who has higher rank, A or B?" What does C answer?