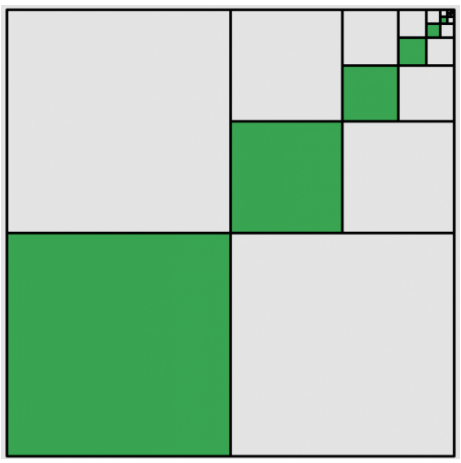
Free the Clones [1]

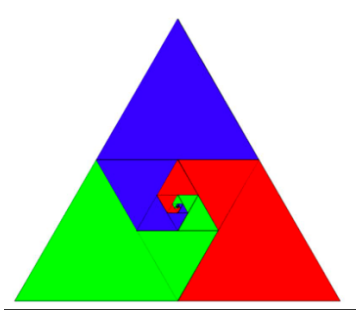
1 Warm-up Problems

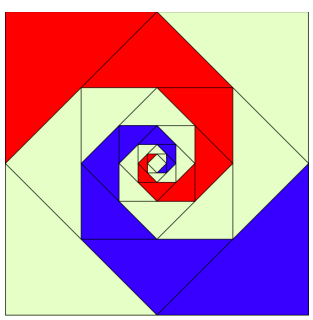
1. Suppose that Homer would like to walk home. In order to do so, he must walk halfway. After he does this, he must walk half of the remaining distance, and then half that distance, etc. Since he will always have half the remaining distance to go, he will never reach home. Logically, we know that Homer can reach home. So how can we resolve this paradoxical situation?

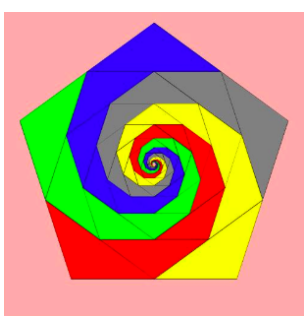
2. If the large square has sides of length 1, then what is the total area of the region that is shaded green?



3. Use each figure to write down an infinite sum and add it up.



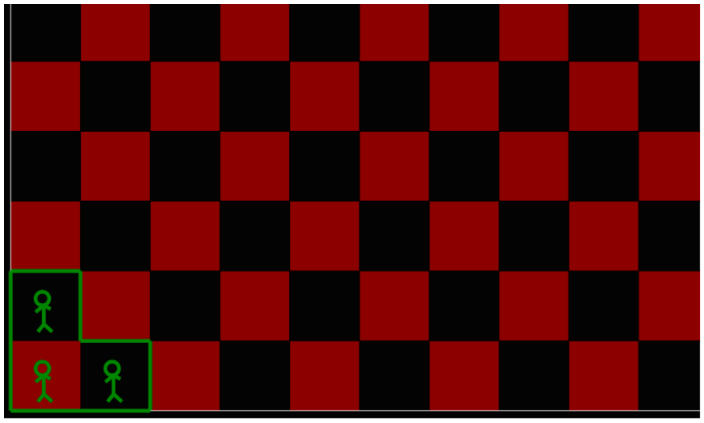




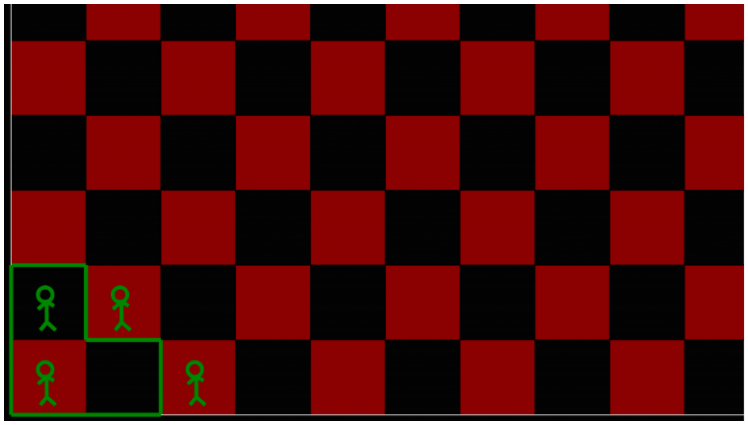
[1] This lesson comes from Zvezda Stankova, via Kimberly Muller's article at Math Teachers Circle website.

2 Free the Clones

4. Imagine that you are standing at the corner of an infinite checkerboard. Three clones are imprisoned in the three tiles at the corner of the board.



The only way for a clone to move is to split itself into two clones, which will occupy the tiles above and to the right of the original clone, as shown below. Only one clone can occupy a tile at a time. For example, if the clone on the right were the first to split, its move would look like this:



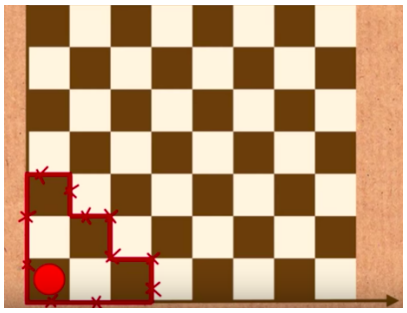
Is it possible to free all the clones from the prison?

See https://openprocessing.org/sketch/126042/ for an app of this game.

See https://www.mathteacherscircle.org/news/mtc-magazine/ws2017/freeing-the-clones/ for a spoiler solution ... but please not until after class!

3 Extra Problems

5. Is the Free the Clones game winnable if we start with one clone? Two clones?
6. Is the Free the Clones games winnable if we start with one clone but a different sized prison? Say our prison has 6 squares instead of 3?



7. What other games can you analyze? Which starting configurations are winnable and which are not?
8. Compute the value of $\frac{1}{1} + \frac{2}{2} + \frac{3}{4} + \frac{4}{8} + \frac{5}{16} + \frac{6}{32} \cdots$

The problems on the last page are from Joshua Zucker and the Julia Robinson Math Festival.