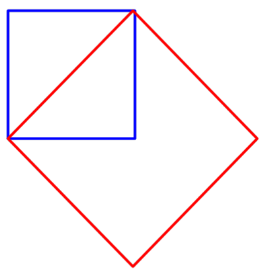
Geometry Puzzles

March 6, 2021

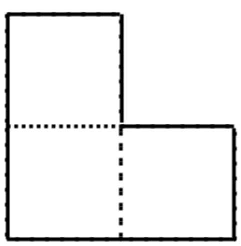
1. If the area of the small square is 1 square cm, what is the area of the big square?[1]



2. The figure shown below can be dissected into three congruent pieces, as shown by the dashed lines. Can you dissect the figure into

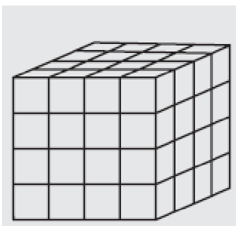
   (a) two congruent pieces?

   (b) four congruent pieces?

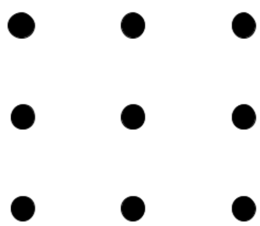


[1] Most of these problems are from *Mathematical Circles: the Russian Experience*

3. A cube with sides n cm long is painted on all faces. It is then cut into cubes with sides 1 cm long. If $n = 4$, the cube looks like the picture below. How many of the smaller cubes will have paint on
   (a) 3 surfaces?
   (b) 2 surfaces?
   (c) 1 surface?
   (d) 0 surfaces?

   What is the answer when $n = 4$? What is the answer for any value of n?
4. Draw 4 straight line segments that pass through all 9 points shown below, without picking up your pencil. (The endpoint of the first line segment must be the starting point of the second line segment, etc.)

5. Cut a square into 5 rectangles in such a way that no two of them have a complete common side (but they may have some parts of their sides in common).
6. Is it true that among any 10 segments, there are always 3 which can form a triangle?
7. A king wants to build 6 fortresses and connect each pair of them by a road. Draw a scheme of fortresses and roads such that there are only 3 points where roads cross each other, and at each of these intersections, there are only two roads crossing each other.
8. Is it possible to choose 6 points on the plane and connect them by disjoint segments (that is, by segments which do not have common inner points) so that each point is connected with exactly 4 other points?
9. Can we tile the plane with congruent pentagons?
10. Is it possible to cut a square into several triangles, all of which are obtuse?
11. Cut a 3×9 rectangle into 8 squares.
12. Prove that a square can be dissected into 2018 squares.
13. Cut an arbitrary triangle into 3 parts such that they can be be rearranged to form a rectangle.

14. Is it possible to draw a closed 8-segment connected broken line which intersects each segment of itself exactly once?