Introduction to Image Processing

Using Linear Algebra to Edit Pictures

CHMC Advanced Session, Feb. 6, 2021

Learning Goals for Today

1. Review matrix arithmetic

2. Use matrices to store image data

3. Use matrix arithmetic to manipulate and edit images

Zoom Protocols
(Same as always)

1. Be respectful of others’ voices and ideas.
2. Whenever possible, have your camera ON during breakout rooms.
3. Change your zoom name to the name you would like to be referred to as.
4. Keep your microphone muted, unless you’re speaking.
5. If you have questions, either raise your hand or use the chat (mainly in the main room).
6. Participate!
7. Think before blurting out an answer, so others have a chance to think.

Review from Last Session

- What are matrices?
- Matrix bookkeeping
  - Shape/dimension
- Matrix algebra
  - Matrix addition
  - Matrix multiplication
  - Scalar multiplication
- Special Matrices
  - Inverse
  - Identity
  - Fibonacci Matrix

Matrix Bookkeeping

- Shape/dimension
- Elements

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Matrix Addition

- Question: When is matrix addition allowed?

$$A + B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b21 & a_{22} + b22 \end{bmatrix}$$

- Question: Is matrix addition commutative?

Matrix Multiplication

- Question: When is matrix multiplication allowed?

$$A \cdot B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} & a_{11} \cdot b_{12} + a_{12} \cdot b_{22} \\ a_{21} \cdot b_{11} + a_{22} \cdot b_{21} & a_{21} \cdot b_{12} + a_{22} \cdot b_{22} \end{bmatrix}$$

- Question: Is matrix multiplication commutative? Why or why not?

Special Matrices

- Question: How are the inverse and identity matrices related?

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Some matrices can be used to encode information, such as the Fibonacci matrix

$$F = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \qquad F^n = F \cdot F \cdot \ldots \cdot F = \begin{bmatrix} F_n & F_{n-1} \\ F_{n-1} & F_{n-2} \end{bmatrix}$$

Another Special Matrix

Transpose Matrix

- Take matrix and move the entries around in a specific way
- Question: Can you tell what has been moved in this matrix?

$$A^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

So why are matrices useful?

- Store information
- Compute specific information

Yearly Use Increase Count (Millions)

2018 2019

TikTok, Snapchat

$$\begin{bmatrix} 271 & 508 \\ 76 & 80 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -271 + 508 \\ -76 + 80 \end{bmatrix} = \begin{bmatrix} +237 \\ +4 \end{bmatrix}$$

Using Matrices to Store Images

- An image is just a matrix where every entry indicates a color
- Various 8 bit color scales
  - Grayscale: [0, 255]
    - 0 = black
    - 255 = white
  - RGB: red, green, blue [0, 255]

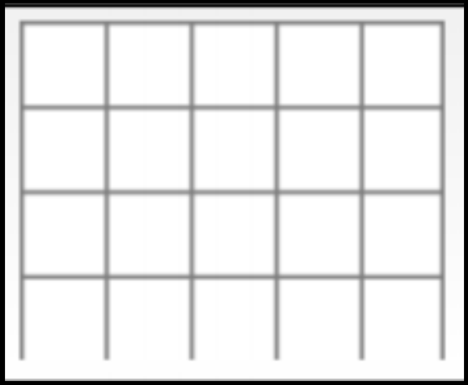
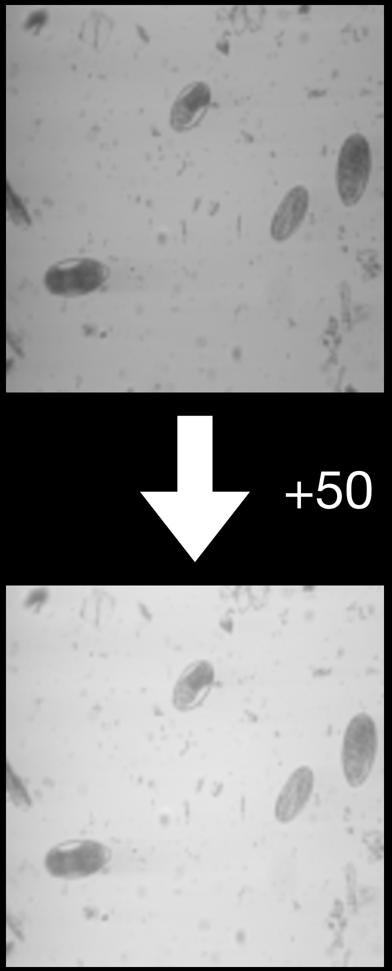


Image Transformations

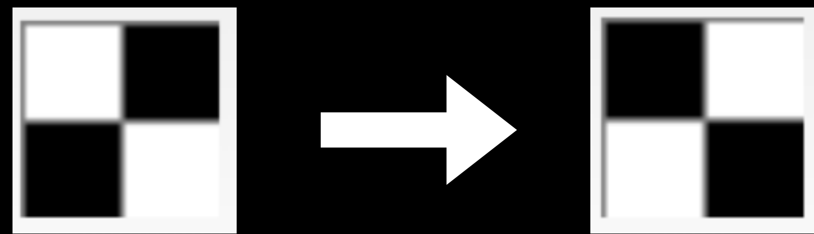
- Matrix addition
  - “Adds” two images together
  - Adds or subtracts a fixed amount from all pixels (constant matrix)
    - Brightening/darkening
- Scalar multiplication
  - Increases or decrease all pixels by the scalar ratio
- Question: What happens when a new pixel is computed with a value outside of [0, 255]?



BREAKOUT ROOMS

Reflecting

- Mapping functions
  - Reflecting over the x-axis
    $f(x) \to -f(x)$
  - Reflecting over the y-axis
    $f(x) \to f(-x)$



$$\begin{bmatrix} 255 & 0 \\ 0 & 255 \end{bmatrix} \qquad \begin{bmatrix} 0 & 255 \\ 255 & 0 \end{bmatrix}$$

- Want a mapping that reflects any image over its x-axis or y-axis
- We want to keep the entries the same, just move them
- Question: what matrix when multiplied with, preserves both the entire and their locations?
- Question: can a non-square matrix be reflected?

BREAKOUT ROOMS

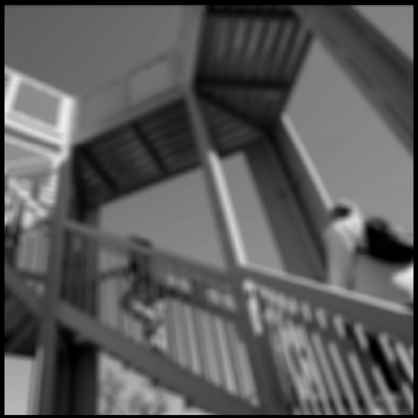
Another Special Matrix

Permutation Matrix or "Reverse-Identity" Matrix

- We have discovered that the matrices $X = Y = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- Where we multiply this matrix will determine which axis we reflect over
- Can we reflect over the origin?
- $$\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} d & c \\ b & a \end{bmatrix}$$
- Question: What is the resultant matrix most similar to?

Blurring Images

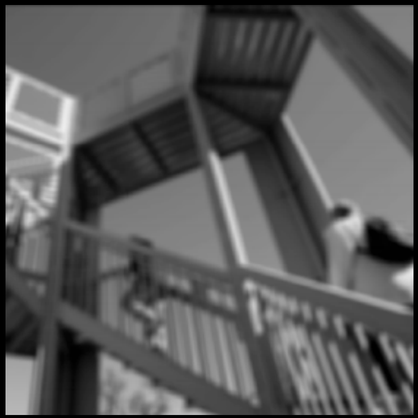
- Question: How can we define blurring mathematically?



$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 3 & 1 & 2 \end{bmatrix} \rightarrow \ ?$$

Blurring Images

- Question: How can we define blurring mathematically?



$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 3 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 5/4 & 8/6 & 5/4 \\ 9/6 & 14/9 & 8/6 \\ 7/4 & 10/6 & 5/4 \end{bmatrix}$$