

Warm-up

In the following “cryptograms” each letter stands for a digit 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. If letter appears twice in the same problem, then it stands for the same digit each time. Two different letters in the same problem stand for different digits.

$$\begin{array}{r} \text{M E} \\ + \text{ M} \\ \hline \text{A S A} \end{array}$$

$$\begin{array}{r} \text{S U P} \\ + \text{S P U} \\ \hline \text{U P S} \end{array}$$

Part 1: Cryptograms¹

In the following “cryptograms” each letter stands for a digit 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. If letter appears twice in the same problem, then it stands for the same digit each time. Two different letters in the same problem stand for different digits.

$$1. \begin{array}{r} \text{A T} \\ \text{A T} \\ + \text{A T} \\ \hline \text{B A T} \end{array}$$

$$\begin{array}{r} \text{B B} \\ \text{A} \\ + \text{A} \\ \hline \text{C C C} \end{array}$$

$$2. \begin{array}{r} \text{A H A} \\ + \text{H} \\ \hline \text{B E E} \end{array}$$

$$\begin{array}{r} \text{O D D} \\ + \text{O D D} \\ \hline \text{U N D O} \end{array}$$

3. Explain why the following puzzles have no solutions:

$$\begin{array}{r} \text{K A T H R I N} \\ + \text{B E L L A} \\ \hline \text{F R I E N D S} \end{array}$$

$$\begin{array}{r} \text{B A T} \\ + \text{R A T} \\ \hline \text{C A T} \end{array}$$

4. Solve these puzzles.

$$\begin{array}{r} \text{A H A} \\ + \text{E H E} \\ \hline \text{A H A H} \end{array}$$

$$\begin{array}{r} \text{R O S A} \\ + \text{R O S A} \\ \hline \text{O A S I S} \end{array}$$

5. SEND + MORE = MONEY

6. Can these puzzles be solved?

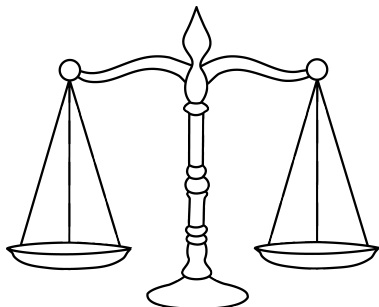
$$\begin{array}{r} \text{C O K E} \\ + \text{C A K E} \\ \hline \text{S O C I A L} \end{array}$$

$$\begin{array}{r} \text{T E E} \\ + \text{I C E} \\ \hline \text{N I C E} \end{array}$$

¹From *Mathematical Circle Diaries* by Ana Burago

Part 2: Coins ²

1. Suppose you have 3 coins, and know that exactly one of them is counterfeit – lighter than the real ones. How can you find the counterfeit coin using a single weighing on a balance scale?



2. Suppose you have 3 coins, and know that exactly one of them is counterfeit – but you don't know whether it is lighter or heavier than the real ones! How can you find the counterfeit coin using just two weighings on a balance scale with two pans?
3.
 - (a) You have 4 coins, and know that exactly one of them is counterfeit and lighter than the real ones. Find the counterfeit coin in two weighings on the balance scale.
 - (b) The same problem for 5 coins and two weighings.
 - (c) The same problem for 7 coins and two weighings
 - (d) The same problem for 9 coins and two weighings.
4. You have 4 coins. One of them is counterfeit – however, you don't know if it is lighter or heavier than the real ones. How can you find the counterfeit coin using just two weighings on a balance scale with two pans?
5.
 - (a) Out of 75 gold coins, one is counterfeit. However, you don't know whether it is lighter or heavier than a real one. Using two weighings on a balance scale, find out whether the counterfeit coin is lighter or heavier than a real one.
 - (b) The same question for 76 coins.