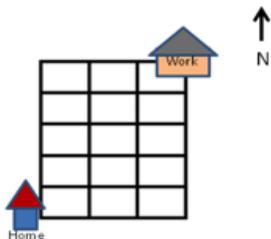


Combinatorics¹

- How many ways are there to arrange the letters in the word HOGWARTS? The letters in the word VOLDEMORT? The letters in the word ALOHOMORA? The letters in the words AVADA KEDAVRA?
- You have 2 Reece's pieces, 4 mini bags of M&M's, and 1 sour candy left from Halloween. You are going to eat one candy per day until the candy is gone. How many different ways can you do this?
- How many ways can you make a pizza with 3 different toppings if there are 8 toppings to choose from? (What if the 3 toppings don't all have to be different? For example, you could have mushroom and double pepperoni as one option.)
- How many different routes are there from home to work, only traveling north and east on streets?



- There are three rooms in a dormitory: one single, one double, and one for four students. How many ways are there to house seven students in these rooms?

Permutations:

Permutations: The number of ways to lay out n different objects in a row is

$$n! = n \cdot (n - 1) \cdot (n - 2) \dots 3 \cdot 2 \cdot 1$$

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Combinations:

The number of ways to choose k objects out of a collection of n objects is

$$\binom{n}{k} = \frac{n \cdot (n - 1) \cdot (n - 2) \dots (n - k + 1)}{k!} = \frac{n!}{k!(n - k)!}$$

- How many ways are there to distribute 10 doggie biscuits among 7 dogs? The biscuits are indistinguishable, but the dogs are distinguishable.
- How many ways are there to represent the number 12 as a sum of
 - 5 non-negative integers?
 - 5 positive integers?

¹Many of these problems are from *Mathematical Circles (Russian Experience)*. Others are from Paul Zeitz.

The order of the numbers matters here, so, for example, $1 + 4 + 5 + 1 + 1$ is considered different from $1 + 1 + 1 + 4 + 5$.

8. You roll a 6-sided die 3 times. Among all possible outcomes, how many have at least one occurrence of the number 6?



9. You flip a coin 10 times. Of all the possible outcomes, how many have exactly 5 heads in a row? For example, we would not count HHHHHHHTTT (too many consecutive heads), but we would count TTTHHHHHTT and HHTTHHHHHT.
10. 10 boys and 9 girls sit in a row of 19 seats. How many ways can this be done if
- (a) All the boys sit next to each other and all the girls sit next to each other.
 - (b) The children sit so that each person has only neighbors of the opposite sex.
11. How many ways can you choose a team from 11 people, where each team must have at least one person and the team must have a designated captain? The team can have 1 or more players including the captain.
12. In a traditional village, there are 10 young men and 10 young women. The village matchmaker arranges all the marriages.
- (a) In how many ways can they pair off the 20 young people, assuming a marriage has to be between a man and a woman?
 - (b) In how many ways can they pair off the 20 young people if male-male couples, male-female couples, and female-female couples are all allowed?
13. All 11 girls in Ms. Jewls' class are going to the circus, and will be sitting in a row of 11 seats. How many ways are there for them to seat themselves if Mauricia and Deedee refuse to sit next to each other?
14. In how many ways can you choose a debate team of 6 students from Ms. Jewls' class of 11 girls and 17 boys? What if the team has to consist of 3 girls and 3 boys?
15. Ms. Jewls' class is going to play capture the flag at recess. How many ways are there to divide the 28 students into 2 teams of 14? What if Maricia and Deedee can't be on the same team?
16. What is the largest number of triangles you can make by drawing 7 lines in the plane? The triangles may overlap or contain each other.

17. How many ways are there to rearrange the letters in the word "FLAMINGO" so that the vowels will be in alphabetical order and so will the consonants? For example, FAGILMON (A - I - O, F - G - L - M - N).
18. Three different flavors of pie are available, and seven children are each given a slice of pie in such a way that at least two children get different flavors. How many ways can this be done?
19. How many ways are there to distribute 7 different prizes (e.g. best hair, best apple pie, etc.) among 26 people,
 - (a) if it is NOT possible for one person to win more than one prize?
 - (b) if it IS possible for one person to win more than one prize?
20. How many ways are there to distribute 7 identical prizes (e.g. all of them are \$50 gift cards to REI) among 26 people,
 - (a) if it is NOT possible for one person to win more than one prize?
 - (b) if it IS possible for one person to win more than one prize?
21. Find the number of integers from 0 through 999999 that have no two equal neighboring digits.
22. How many ways are there to distribute 3 one-dollar bills and 10 quarters among 4 children?
23. Thirty people vote for 5 candidates. How many possible distributions of their votes are there, if each of them votes for one candidate, and we consider only the number of votes given to each of the candidates?
24. In how many ways can three people divide among themselves six identical apples, one orange, one plum, and one tangerine (without cutting any fruit)?
25. How many ways are there to arrange 5 blue balls and 10 white balls in a row so that no two blue balls lie next to each other?
26. How many ways are there to arrange 5 blue balls, 5 red balls, and 5 green balls in a row so that no two blue balls lie next to each other?
27. There are 12 books on a shelf. How many ways are there to choose five of them so that no two of the chosen books stand next to each other?
28. How many ways are there to choose 6 cards from a complete deck of 52 cards in such a way that all four suits will be present?

29. Consider the following poker hands:

- (a) Royal flush: A, K, Q, J, 10, all the same suit.
- (b) Straight flush: Five cards in a sequence, all in the same suit.
- (c) Four of a kind: All four cards of the same rank.
- (d) Full house: Three of a kind with a pair.
- (e) Flush: Any five cards of the same suit, but not in a sequence.
- (f) Straight: Five cards in a sequence, but not of the same suit.
- (g) Three of a kind: Three cards of the same rank.
- (h) Two pair: Two different pairs.
- (i) Pair: Two cards of the same rank.

Find the probability of each kind of hand.