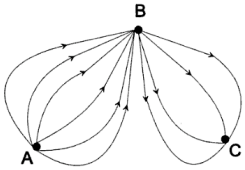
Combinatorics[1]

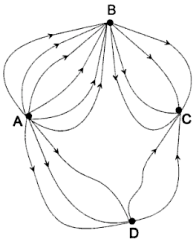
1. There are eight different flavors of ice cream and three different types of cones at the Yogurt Pump counter. You'd like to buy a different type of ice cream cone every day. How many days can you do this before you have to repeat?

2. There are also 4 types of toppings. Now how many ways are there to buy a cone, ice cream, plus topping before you have to repeat?

3. There are three towns A, B, C, on Frog Island. There are 6 roads from A to B and 4 roads from B to C. How many ways can you drive from A to C (without going back through A!)



4. A new town D is built and 5 new roads as shown. Now how many ways are there to drive from A to C?



5. A garden store has 4 types of cucumber plants, 7 types of tomato plants, and 3 types of watermelon. How many ways can you pick 2 different types of plants to put in your garden, if you don't want them both to be cucumbers, or both tomatoes, or both watermelons?

Multiplication vs. Addition:

(a) If we are counting the number of outcomes of a multi-stage process (like choosing from a sequence of menus), then the number of outcomes is the product of the number of choices for each stage.

(b) Whenever we separate the outcomes into several cases, each requiring separate counting methods, we add the number of outcomes in each case to get the total number of outcomes.

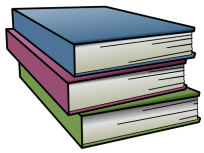
(c) Sometimes its easiest to count the number of outcomes we don't want and subtract that number from the total number of outcomes.

[1]Many of these problems are from *Mathematical Circles (Russian Experience). Others are from Paul Zeitz.*

6. We call a natural number ultra-odd if all of its digits are odd. How many four-digit ultra-odd numbers are there? How many four-digit ultra-even numbers are there (i.e. all digits even)?

7. The Braille writing system uses 6 dots to represent the 26 letters of the English alphabet. Each dot can be either raised or flat. Are 6 dots also enough to represent the Japanese Katakana syllabic writing system, with 46 symbols?

8. There are 5 books on a shelf. How many ways are there to arrange three of them in a stack?



9. There are 11 girls and 17 boys in Ms. Jewls' class. How many ways are there to choose one boy and one girl to represent the class in a math contest? How many ways are there to choose 2 students of the same gender?

10. A code alphabet has only three letters: A, B, and C. A message in this code is an arbitrary sequence of no more than four letters, such as "CBB", or "ABAC". How many messages are possible?

11. Joan has 6 dogs. In how many ways can she choose one or more of them to accompany her on a hike?

12. You roll a 6-sided die 3 times. Among all possible outcomes, how many have at least one occurrence of the number 6?

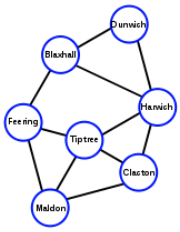
13. How many five-digit numbers have an even sum of their digits?

14. How many even three-digit numbers have no repeating digits?

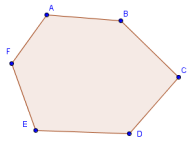
15. How many four-digit numbers have one or more repeating digits?

16. You flip a coin 10 times. Of all the possible outcomes, how many have exactly 5 heads in a row? For example, we would not count HHHHHHHTTT (too many consecutive heads), but we would count TTTHHHHHTT and HHTTHHHHHT.

17. There are 20 towns on Planet X and every pair of them is connected by a road. How many roads are there? (There are no other roads besides the ones that connect the towns.)



18. How many diagonals are there in a convex n-gon (a polygon with n-sides)? A diagonal is a line segment that connects two vertices that are not adjacent.



19. 10 boys and 9 girls sit in a row of 19 seats. How many ways can this be done if

    (a) All the boys sit next to each other and all the girls sit next to each other.

    (b) The children sit so that each person has only neighbors of the opposite sex.

20. How many ways can you choose a team from 11 people, where each team must have at least one person and the team must have a designated captain?

21. There are 6 boys and 6 girls in a dance class.

    (a) In how many ways can they partner off into boy-girl couples?

    (b) In how many ways can they partner off into couples if boy-boy couples, boy-girl couples, and girl-girl couples are all allowed?

22. How many ways are there to arrange the letters in the word HOGWARTS? The letters in the word VOLDEMORT? The letters in the word ALOHOMORA? The letters in the words AVADA KEDAVRA?

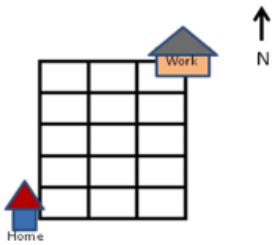
23. You have 2 Reece's pieces, 4 mini bags of M&M's, and 1 sour candy left from Halloween. You are going to eat one candy per day until the candy is gone. How many different ways can you do this?

24. All 11 girls in Ms. Jewls' class are going to the circus, and will be sitting in a row of 11 seats. How many ways are there for them to seat themselves if Mauricia and Deedee refuse to sit next to each other?

25. In how many ways can you choose a debate team of 6 students from Ms. Jewls' class of 11 girls and 17 boys? What if the team has to consist of 3 girls and 3 boys?

26. How many ways can you make a pizza with 3 different toppings if there are 8 toppings to choose from? (What if the 3 toppings don't all have to be different? For example, you could have mushroom and double pepperoni as one option.)

27. How many different routes are there from home to work, only traveling north and east on streets?



Permutations:
Permutations: The number of ways to lay out n different objects in a row is

$$n! = n \cdot (n-1) \cdot (n-2) \ldots 3 \cdot 2 \cdot 1$$

.
Combinations:
The number of ways to choose k objects out of a collection of n objects is

$$\binom{n}{k} = \frac{n \cdot (n-1) \cdot (n-2) \ldots (n-k+1)}{k!} = \frac{n!}{k!(n-k)!}$$

28. Ms. Jewls' class is going to play capture the flag at recess. How many ways are there to divide the 28 students into 2 teams of 14? What if Maricia and Deedee can't be on the same team?

29. What is the largest number of triangles you can make by drawing 7 lines in the plane? The triangles may overlap or contain each other.

30. How many ways are there to rearrange the letters in the word "FLAMINGO" so that the vowels will be in alphabetical order and so will the consonants? For example, FAGILMON (A - I - O, F - G - L - M - N).

31. How many ways are there to distribute 10 doggie biscuits among 7 dogs? The biscuits are indistinguishable, but the dogs are distinguishable.

32. How many ways are there to represent the number 12 as a sum of

    (a) 5 non-negative integers?

    (b) 5 positive integers?

    The order of the numbers matters here, so, for example, 1 + 4 + 5 + 1 + 1 is considered different from 1 + 1 + 1 + 4 + 5.