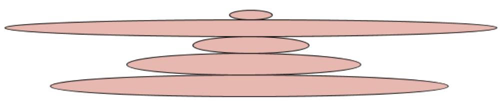
1 Pancake Flipping

Harry Dweighter is a waiter navigating a busy restaurant with a stack of pancakes, all of different sizes. To avoid disaster, Harry wants to sort the pancakes in order by size, with the biggest pancake on the bottom. Having only one free hand, the only available operation is to lift a top portion of the stack with a spatula, turn it upside down, and replace it. Harry wants to figure out how many times will he have to flip to get the pancakes in order, if he flips as efficiently as possible.

1. Find the minimum number of flips needed to reorder this stack of 5 pancakes.



   Harry Dweighter wants to make sure he can deliver the pancakes before they get cold, so he would like to know the worst-case-scenario number of flips he'd have to perform on any given number n of pancakes. We'll call this the pancake number $P(n)$.

   For example, for two pancakes, he might need 0 flips, if the pancakes happen to be in order already, but he might need 1 flip. He won't need any more than 1 flip. So for two pancakes, the pancake number $P(2) = 1$.

2. For three pancakes, what are all the possible ways of arranging the pancakes? For each way, find the minimum number of flips needed to order this stack.

3. What is $P(3)$?

4. What is $P(4)$ and which order(s) of four pancakes take the most flips to put in order?

5. Try to find an algorithm (a systematic method, or set of instructions for the waiter) that will always get any stack of pancakes in order. How many flips will your algorithm take, worst case, for a stack of 4 pancakes? 5 pancakes? 6 pancakes? 7 pancakes? n pancakes?

6. Find an upper bound on $P(n)$: the number of flips you need, worst case, for n pancakes. Your answer should be in terms of n.

7. Find a lower bound on $P(n)$. Remember, $P(n)$ is the number of flips needed to efficiently order a stack of n pancakes, in the worst case scenario for how the pancakes are originally ordered. To find a lower bound. try to find a tricky stack of pancakes for each size n that will take a lot of flips. Again, your answer should be in terms of n.

8. If the pancakes are burnt on one side, then the stack that gets sent out to the customer should not only be in order, but it should also have all the burnt sides facing down. What is the worst-case number of flips $B(n)$ for two, three, and four burnt pancakes?

9. Can you find an upper bound on the burnt pancake number $B(n)$?

10. Can you find a lower bound on $B(n)$?

2 Chicken Nuggets

1. In the U.K., Chicken Nuggets are sold in "ShareBox" packs of 6, 9 or 20. Is it possible to order exactly 30 chicken nuggets? What about 37? 38?

2. What is the largest number of chicken nuggets that you CANNOT buy in the U.K. ?

3. A Happy Meal in the U.K. includes a pack of 4 nuggets. So if you are willing to buy Happy Meals, you can get packs of 4, 6, 9, or 20. Now what is the largest number of unpurchasable chicken nuggets?

4. In the U.S., chicken nuggets are sold in packs of 4, 6, 10, 20 and 50. What is the a largest number that of nuggets that can't be purchased in the U.S.?

5. If nuggets come in packs of size x and y, then what has to be true about the numbers x and y for there to be a largest number that can't be purchased?

6. If chicken nuggets come in packs of size x and y, we'll call the largest unpurchasable number the "chicken nugget number" of x and y and write it as $C(x, y)$.

   Experiment with different values of x and y, and find $C(x, y)$.

   It may help to write out the numbers in rows of length x, where x is the smaller of the two numbers. For example, if x and y are 5 and 9, you could write out the numbers like this:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50
51	52	53	54	55

   Then cross out purchasable numbers to see what is left.

7. Bay Area Rapid food sells chicken nuggets. You can buy packages of 11 or 7. What is the largest integer n such that there is no way to buy exactly n nuggets? Can you Generalize ?

8. Suppose a football league has only scores of field goals (3 points) and touchdowns with the extra point (7 points). Then what is the greatest score that cannot be the score of a team in this football game (ignoring time constraints)?

9. In the National Football League, the only way for a team to score exactly one point is if a safety is awarded against the opposing team when they attempt to convert after a touchdown. Two points are awarded for safeties from regular play, and 3 points are awarded for field goals. What pairs of scores for the two teams are possible and which are impossible (ignoring time constraints).

10. Suppose the envelope can hold ONLY three stamps, and the available stamp values are 1 cent, 2 cents, 5 cents, and 20 cents. What is the SMALLEST amount of postage that cannot be placed on an envelope?

11. Ninety-four bricks, each measuring $4 \times 10 \times 19$ inches are to stacked one on top of another to form a tower 94 bricks tall. Each brick can be oriented so it contributes 4 inches or 10 inches or 19 inches to the total height of the tower. How many different tower heights can be achieved using all ninety-four of the bricks?

12. Find the sum of all positive integers n such that, given an unlimited supply of stamps of denominations $5, n$, and $n + 1$ cents, 91 cents is the greatest postage that cannot be formed.