The Kelin Bottle

Most of this material is from *The Shape of Space* by Jeff Weeks. The word searches were taken from Torus Games at http://geometrygames.org/TorusGames/index.html. You can download the Torus Games app for many other games.

1 The Klein Bottle

1. What surface do you get when you glue together the sides of the square as shown?

![Square with glued sides](image1)

2. What happens as this creature travels through its Klein bottle universe?

![Klein bottle with a path](image2)

- A path that brings a traveler back to his starting point mirror-reversed is called an orientation-reversing path.
- A surface that contains an orientation-reversing path is called non-orientable.

3. Can you find more than one orientation reversing path in this surface?
2 Tic-Tac-Toe on a Klein Bottle

4. Which of these are winning positions in Klein bottle Tic-Tac-Toe?

5. Where can X go to win immediately in Klein bottle Tic Tac Toe?

6. Play a few rounds of Klein bottle Tic-Tac-Toe with a classmate.
   • Is there a winning strategy?
   • Is it possible to get a cat’s game?
   • How many essentially different first moves are there?

7. What are the best moves for X in these positions?
8. A ladybug on a Klein bottle walks in a straight line until she returns to her starting point. She walks 1 unit northward for every 1 unit eastward. Draw her path.

9. Try this word search on the Klein bottle. The arrows show how the sides are glued together.

10. What happens when you cut a Klein bottle in half? Hint: It depends on how you cut it.
3 3-manifolds

11. Is there a 3-dimensional analog to the torus? What about the Klein bottle?