Intermediate, Math Circles, UNC Chapel Hill **November 2, 2018**

(adapted from a handout by Joshua Zucker)

Soma Cube

The Soma Cube was invented by Piet Hein in the 1930's. It has seven pieces. Seven is the number of ways 3 or 4 cubes can be joined face-to-face, so that the resulting shape is NOT rectangular.

1. Try to put the 7 pieces together to form a cube. Don't try too long! Look at the problems and hints below.

2. Look at the pieces. How many cubes are in each piece?

   How many different pieces could you make with 3 cubes? (How many of these would be part of the Soma Cube?)

   How many different pieces could you make with 4 cubes? (How many of these would be part of the Soma Cube?)

   If you are curious, how many different pieces could you make with 5 cubes?

3. Think about the problem of covering an 8 by 8 chessboard with dominoes. What if you removed 2 opposite corners of the chessboard? Would you be able to cover the modified chessboard with dominoes? Why or why not?

4. Now, think about a 3x3x3 cube (the Soma Cube).

   How many individual cubes would it take to make the Soma Cube?

   Suppose we use black and white cubes and put them together in such a way that black cubes must be next to white cubes, and, of course, white cubes must be next to black cube. Are all the corners the same color?

   What about the edges? What about the centers of each face? What about the center of the 3x3x3 cube?

   Assume that one corner is made from a black cube. Then, how many white cubes and how many black cubes will we need to build the 3x3x3 cube?

5. Imagine the 3x3x3 Soma Cube – How many vertices (corners) does it have?

6. Look at the pieces of the Soma Cube. How many corners of the 3x3x3 cube can each piece touch? What is the maximum number and the minimum number of corners that each piece can touch? (You might want to make a table of values.)

7. Look at each piece, one at a time. If you make the pieces with black and white cubes, checkerboard style, which pieces are made from an equal number of black and white cubes and which are not?

8. Is it possible for the T piece to fill just one corner? Can it fill NO corners in the 3x3x3 cube? Where must it go in the 3x3x3 cube?

9. If the T piece has to go on an edge, where can the Y piece go? (The Y piece is the one made of 4 cubes that looks like a corner, and is one that requires 3 dimensions.)

10. The V piece, made of just 3 pieces, might be the easiest piece to place, so save that for last. Think about whether it will be made of 2 black and 1 white or the other way around. (Assume that the corner color is black.)

11. Can you make a table to show where each piece could go in the 3x3x3 cube?

12. Can you guess how many solutions there are in total for the Soma Cube? Can you figure out an upper-bound for the number of solutions?

13. There are MANY ways to dissect a cube into pieces like the pieces of the Soma Cube, giving many possible puzzles. Some are more difficult than Soma and some are easier. Some have a single solution, while others have many solutions. (The Soma Cube actually has 240 different solutions?!)

14. You can also use the pieces of the Soma Cube to make shapes other than a 3x3x3 cube.

    Can you make a 3x3x3 cube with a bottom corner missing, and an extra cube on top of a top corner?

    Can you make a 3x3x3 cube with a bottom edge piece missing, and an extra cube on top of a top edge?

    Can you have 2 missing bottom edge pieces, and 2 extra pieces on top of top edges?