

Counting and Symmetry¹

1. How many ways are there to put 6 different beads in order? n different beads?
2. What if the beads are in a circle, so rotations don't matter?
3. What if the beads are on a bracelet, which can be rotated or flipped over?
4. Redo problem 1 through 3 with w identical white beads and b identical black beads. Hmm, maybe that's too hard. Start with one white and one black, and work your way up. Can we solve it with 14 white and 6 black beads? Would it be much easier with 5 black instead of 6?

See how much harder it gets when there's symmetry?

5. How many ways are there to paint a six-sided die if you have one color of paint? Two colors? Three colors? n colors? Note that painting the side with one dot red and the side with two dots blue is different from painting the side with one dot blue and the side with two dots red. You have to paint all the sides.
6. Now try the much harder problem: painting a cube with six identical faces.
7. How many different ways are there to put six identical beads around a bracelet? What if there are 5 of one color and 1 of another? Or 4, 2? Or 4, 1, 1? How about 3, 2, 1? How about all the ways using at most 3 different colors?
8. A classic tie-dye shirt problem: how many ways are there to make a shirt with k vertical stripes if you have n colors to choose from? What if the front of the shirt and the back of the shirt are indistinguishable and each stripe soaks through both the front and the back? It is allowable to have two or more adjacent stripes the same color.
9. Let's back up to some easier problems: how many ways are there to color the sides of a triangle with k colors? (Four possible answers: with no symmetry, rotation only, reflection only, or rotation and reflection.)
10. Same question, with a square.
11. How about an n -gon?
12. How many graphs are there on n vertices? (Start with small numbers.)

¹These problems are from Joshua Zucker, with thanks also to Tom Davis. See the article [polya.pdf](http://www.geometer.org/mathcircles/index.html) at <http://www.geometer.org/mathcircles/index.html>.