Math Circle Worksheet:
Topology through Play-Doh

10/6/18

1 Warm-up problems

Exercise 1.1 Take your play-doh, work with it.

Exercise 1.2 Form the letter "A" with your play-doh. Without tearing your play-doh, morph the "A" into an "O". How did you carry out the morphing?

Exercise 1.3 Now take the letter "L". Show that this letter can be morphed into a ball.

Exercise 1.4 Is it possible to morph "A" into "L"? Why or why not?

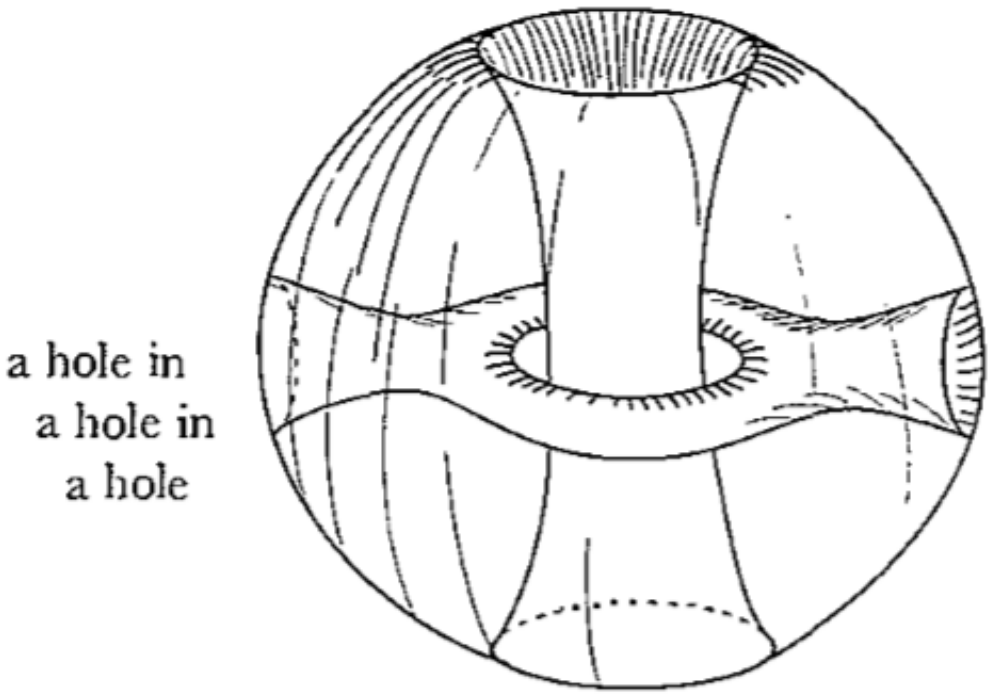
Exercise 1.5 Is it possible to morph the letter "A" into the letter "B"? Again, you're not allowed to tear the play-doh.

Exercise 1.6 We say that "A" and "O" are **equivalent** because they can be morphed into one another, whereas "A" and "B" are not equivalent. What are all of the letters equivalent to "A"? What are all of the letters equivalent to "B"? What letters are left?

Exercise 1.7 What characteristics do the letters in the "A" group all share? What about the other two groups?

Exercise 1.8 What would a shape with 3 "holes" look like? What about 4 holes? Construct some with play-doh. Is it possible to morph the 3-holed shape into the 4-holed shape you constructed?

Exercise 1.9 *How many "holes" does the following shape (a hole in a hole in a hole) have? Why? Hint: construct this shape using play-doh, and try to morph it into something where you can easily identify the number of holes.



Exercise 1.10 *What would a hole in a hole in a hole in a hole look like? How many holes are there in total? How do you know?

2 Classifying surfaces

Exercise 2.1 Take your play-doh and form a square. Take the top and bottom edges and combine (glue) them. Next, take the left and right circles and combine them. What shape do you get? (This shape is called a torus.)

Exercise 2.2 How would you glue two tori together to get a 2-holed shape that is equivalent to a "B"?

Exercise 2.3 Before we used to get a square to form a torus. How might you use an octagon to form a shape equivalent to a "B"? Hint: each of the original tori in the "B" comes from a square. When you're combining the two tori, what happens to each square?

Exercise 2.4 Now combine three tori to get a three-holed torus. You should get something that looks like a fidget spinner. What kind of polygon would you use to form this shape?

Exercise 2.5 In general, if you want to make an n-holed torus from a polygon, how many sides does the polygon need to have?

3 A clothesline puzzle

Exercise 3.1 Take a ball, and form a "Y" shaped hole in the center. How many holes does the resulting shape have? Why? (2.)

Exercise 3.2 Morph the ball with the "Y" shaped hole into a 2-holed torus. How many different ways can this be done?

Exercise 3.3 You hang a 2-holed torus up by stringing a clothesline through one hole, and tying the ends of the clotheslines up to two different walls. It is possible to morph this 2-holed torus with a string passing through one hole, into a 2-holed torus with the string passing through both holes; how? (The above two exercises are not unrelated.)

4 Mobius strip fun

Exercise 4.1 Take a square of play-doh. Take the left and right edges, and glue them together; this shape is called a band. If you cut the band along the middle line, how many objects do you end up with?

Exercise 4.2 Take a square of play-doh. Take the left and right edges, morph the play-doh so that one edge is upside-down (give one edge a twist), and now glue the edges together. This shape is called a **Mobius band**. Now cut this shape along the middle line. How many objects do you end up with?

Exercise 4.3 Do the same construction as above, only this time give the play-doh two twists. Cut the shape along the middle line; how many objects are you left with? What is different in this case?

Exercise 4.4 Generalize the above few questions: if you twist the play-doh n times before gluing opposite edges, and then cut along the center line, how many objects will you end up with?