Constructions

April 7, 2018

1. There are two egg timers: one for 7 minutes and one for 11 minutes. We must boil an egg for exactly 15 minutes. How can we do that using only these timers?
2. There are two buttons inside an elevator in a building with 20 floors. The elevator goes 13 floors up when the first button is pressed, and 8 floors down when the second one is pressed (a button will not function if there are not enough floors to go up or down). How can we get to the 8th floor from the 13th?
3. The number 458 is written on a blackboard. It is allowed either to double the number on the blackboard or to erase its last digit. How can we obtain the number 14 using these operations?
4. Cards with the numbers 7, 8, 9, 4, 5, 6, 1, 2, and 3 are laid in a row in the indicated order. We are allowed to choose several consecutive cards and rearrange them in the reverse order. Is it possible to obtain the arrangement 1, 2, 3, 4, 5, 6, 7, 8, 9 after three such operations?
5. The numbers 1 through 16 are placed in the boxes of a 4×4 table as shown in Figure (a). We are allowed to to increase all the numbers in any row by 1 or decrease all the numbers in any column by 1. Is it possible to obtain the table shown in Figure (b) using these operations?

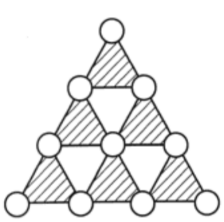
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

(a)

1	5	9	13
2	6	10	14
3	7	11	15
4	8	12	16

(b)

6. Is it possible to write the numbers 1 through 100 in a row in such a way that the (positive) difference between any two neighboring numbers is not less than 50?
7. Divide a set of stones which weigh $1g, 2g, 3g, \ldots, 555g$ into three heaps of equal weight.
8. Fill the boxes of a 4×4 table with non-zero numbers so that the sum of the numbers in the corners of any 2×2, 3×3, or 4×4 square is zero.
9. Is it possible to label the edges of a cube using the numbers 1 through 12 in such a way that the sums of the numbers on any two faces of the cube are equal?
10. Is it possible to place the numbers 0 through 9 in the circles in Figures 59 without repetitions so that all the sums of the numbers in the vertices of the shaded triangles are equal?



11. Prove that it is possible to cross out several digits at the beginning and several at the end of the 400-digit numbers 84198419...8419 in such a way that the sum of the remaining digits is 1984.

12. Find a two-digit number, the sum of whose digits does not change when the number is multiplied by any one-digit number.

13. Do there exist two consecutive natural numbers such that the sums of their digits are both divisible by 7?

14. Do there exist several positive numbers, whose sum is 1, and the sum of whose squares is less than 0.01?

15. A castle consists of 64 identical square rooms, having a door in every wall and arranged in an 8×8 square. All the floors are colored white. Every morning a painter walks through the castle recoloring floors in all the rooms he visits from white to black and vice versa. Is it possible hat some day teh rooms will be colored as a standard chessboard is?

16. Can one place a few dimes on the surface of a table so that each coin touches exactly three other coins?