

## Two is Better than Infinity

### 1 Warm-Up

1. On the Island of Knights and Knaves, knights always tell the truth and knaves always lie.

Island resident Jeremiah says, "Yesterday I baked 39 cookies for my party. Every child at the party ate exactly 4 cookies and every adult ate exactly 2 cookies. Nobody else ate any cookies, not even me! And none were thrown away. By the end of the party, all the cookies were eaten. Too bad I have no cookies left to share with you."

Can you tell if Jeremiah a knight or a knave?

### 2 A Game with Cups

2. With a partner, arrange 7 cups to look like this, with four cups turned upside-down and three cups turned right-side-up.



You are allowed to turn cups over two at a time.

- Two upside-down cups can both be flipped over and placed back on the table right-side-up.
- Two right-side-up cups can both be flipped over and placed back on the table upside-down.
- A pair with one right-side-up and one upside-down cup can be flipped so that they are upside-down and right-side up, respectively.

It is against the rules to flip just one cup or more than two cups at the same time.

You win the game if you can bring all 7 cups to the upside-down position.

### 3 Evens and Odds

3. What does it mean for a number to be even?
4. What does it mean for a number to be odd?
5. What happens when you add an even number and an even number?

6. Fill in the chart by writing "Even", "Odd", or "Can't Tell" .

Even	+	Even	=
Even	+	Odd	=
Odd	+	Even	=
Odd	+	Odd	=
Even	×	Even	=
Even	×	Odd	=
Odd	×	Even	=
Odd	×	Odd	=

## 4 More Problems

7. Louis has reprogrammed the elevator in the 30 story Wayside School. Only two buttons work and the first button sends the elevator 8 floors up and the second one 6 floors down. (The elevator will not move if it is asked to go above the 30th floor or below the 1st floor.)
- (a) Principal Kidswatter is drinking coffee on the first floor. Can he take the elevator to the 23 floor? If so, show how. If not, explain why.
- (b) Can he take the elevator to the 24 floor? If so, show how. If not, explain why.

\*Wayside School experts, please ignore the fact that there is no 19th floor.



8. Harry Potter's copy of the book *Advanced Potion-Making* was falling apart. When he shook it, 25 pages fell out. He used a spell to add up all the page numbers on both sides of each page, and found that the total of all these numbers was 2000.

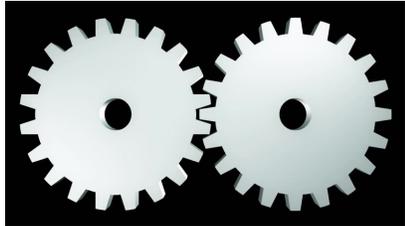
"Harry," said Hermione, "You're doing the spell all wrong."

How did Hermione know?

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This week's problems are from *Mathematical Circle Diaries, Year 1* by Anna Burago.

9. Max placed 11 interlocking gears in a row. Each gear, except for the first and last, is connected to its right and left neighbors. Max turns the leftmost gear clockwise. Is the rightmost gear going to rotate clockwise or counterclockwise?



10. Bella placed her 11 interlocking gears in a circle, in such a way that every gear is connected to two neighbors. She turns the first gear clockwise. What happens then?
11. Kolya placed a full set of dominos (all 28 pieces) into a line according to the rule of dominos (see below). The left half of the leftmost domino has 6 spots on it. How many spots are there on the right half of the rightmost domino?

*Rule of Dominos: When two dominos are placed next to each other, end to end, the halves that are next to each other must have the same number of spots.*

