Problems about Integers, Part 2

Feb 10, 2018

1 Word Problems

1. Two teams played each other in a decathlon. In each event, the winning team gets 4 points and the losing team gets 1 point, and both teams get 2 points in case of a draw. After all 10 events, the two teams have 46 points together. How many draws were there?[1]

   Answer: 4

2. If every boy in a class buys a muffin and every girl buys a sandwich, they will spend one cent less than if every boy buys a sandwich and every girl buys a muffin. We know that the number of boys in the class is greater than the number of girls. Find the difference.

   Answer: 1

3. 175 kiwis cost more than 126 passionfruit. Prove that you cannot buy three kiwis and one passionfruit for one dollar.

4. In a class, every boy is friends with exactly three girls, and every girl is friends with exactly two boys. It is known that there are only 19 desks (each holding at most two students), and 31 of the students in the class study French. How many students are there?

   Answer: 35

5. Four friends bought a boat. The first friend paid half of the sum paid by the others; the second paid one third of the sum paid by the others; the third paid one quarter of what was paid by the others; and the fourth paid 130 dollars. What was the price of the boat, and how much did each of the friends pay?

   Answer: 600

6. The road connecting two mountain villages goes only uphill or downhill (never flat). A bus always travels 15 mph uphill and 30 mph downhill. Find the distance between the villages if it takes exactly 4 hours for the bus to complete a round trip.

   Answer: 40

[1] From Mathematical Circles, the Russian Experience

2 Guess the Number

7. Given that p, $p+10$, and $p+14$ are prime numbers, find p. Hint: find remainders when divided by 3.
8. Given that p, $2p+1$, and $4p+1$ are prime numbers, find p.
9. If p, $4p^2+1$, and $6p^2+1$ are prime numbers, find p.
10. Find the smallest natural number which has a remainder of 1 when divided by 2, a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, a remainder of 4 when divided by 5, and a remainder of 5 when divided by 6.
11. The prime numbers p and q and the natural number n satisfy the following equality:

$$\frac{1}{p} + \frac{1}{q} + \frac{1}{pq} = \frac{1}{n}$$

    Find the numbers.
12. Prove that there is a natural number n such that the numbers $n+1, n+2, n+3, \cdots n+2017$ are all composite.