Probability, Part 2

1 Warm-up

1. Three cards are dealt from a well-shuffled deck.
   (a) Find the chance that all of the cards are diamonds.
   (b) Find the chance that none of the cards are diamonds.
   (c) Find the chance that the cards are not all diamonds.
   (d) Find the chance that at least one card is a diamond.
2. *Random Trisection.* A deck of cards is randomly cut into three piles. I bet that at least one of the cards on the top of a pile is a "face card" i.e. a Jack, Queen, or King. Do you want to take this bet?

2 Duels

3. *Heads I Win.* Two players alternately toss a penny, and the one that first tosses heads wins. What is the probability that
   (a) the game never ends?
   (b) the first player wins?
   (c) the second player wins?
4. *Three-way duel.* Alexander Hamilton, Aaron Burr, and Thomas Jefferson fight a 3-cornered pistol duel. All three know that Alexander Hamilton's chance of hitting any target is 1/3, while Aaron Burr *never* misses, and Thomas Jefferson has a 0.5 chance of hitting any target. The way the duel works is that each person is to fire at their choice of target, starting with Alexander Hamilton, and proceding to Aaron Burr, then Thomas Jefferson, then Alexander Hamilton again, etc. (unless someone is hit, in which case they don't shoot), continuing until one person is left unhit. What is Alexander Hamilton's strategy?

   Experiment with different strategies, and simulate the shooting using dice. You can simulate an event with probability 1/3 by tossing one die and seeing if the number shown is 1 or 2, say. Likewise, you can come up with ways to simulate a 1/2 probability event (you don't need a coin, you can still use a die).

   Here are some possible strategies for Alexander Hamilton: shoot at Aaron Burr first; shoot at Thomas Jefferson first; try something else. Choose a strategy, and try simulating the duel. See if you can experimentally estimate the probability that Alexander Hamilton survives, using various strategies.

   Try to justify your experimental conclusions with calculations.