Pigeonhole Problems

1.   Can one find 4 integer numbers such that the sum and the product of these numbers are both odd?

2.   A high school is struggling to come up with football(11 players), basketball(5 players), baseball(10 players), and volleyball(6 players) teams. Students are free to choose which team they want to be on(only one team per student). How many students need to sign up so that at least one team will be definitely complete(the minimum)?

3.   At a party 20 people shook hands. Nobody shook hands with himself and no two shook hands twice. Prove that there are at least 2 people who shook hands the same number of times.

4.   A contractor needs to create a team of 9 carpenters, a team of 3 electricians, and a team of 4 plumbers. Each available worker can perform only one kind of job. What is the minimal number of workers the contractor needs to choose from to guarantee that at least one team is complete?

5. 102 animals(cats and dogs) broke into McDonalds and ate 506 burgers. Each cat ate 5 burgers, and each dog ate 6 burgers. How many cats and dogs were there?

6.   Show that in any group of $n$ people, there are two who have an identical number of friends within the group.

7.  Six distinct positive integers are randomly chosen between 1 and 2006, inclusive. What is the probability that some pair of these integers has a difference that is a multiple of 5?

8.  Prove that among any ten points located on a circle with diameter 5, there exist at least two at a distance less than 2 from each other.

9.  Prove that from any set of one hundred whole numbers, one can choose either one number which is divisible by 100, or several numbers whose sum is divisible by 100.

10.  You want a fruit basket to contain either at least 8 apples, or at least 5 bananas, or at least 7 pears. What is the smallest number of pieces of fruit you need to choose from to guarantee this?

11.  You have 5 bags with gold coins that look the same. 4 bags have real coins, 1 bag has fake coins. Real coins weigh 10 grams each, but fake coins weigh 11 grams each. The pirates have a scale that can tell you the exact weight of coins, but it can only be used once.