

Feb. 11, 2017

Math Class in the Land of Knights and Knaves

In the land of knights and knaves, knights always tell the truth and knaves always lie. Decide who is a knight and who is a knave.

1. Maurice says, I have two numbers X and Y that are both divisible by 7, but their sum $X + Y$ is NOT divisible by 7.
2. Nina says, I have two numbers P and Q and their difference $P - Q$ is divisible by 7 but neither P nor Q is divisible by 7.
3. Omar says, I have two numbers A and B , and they are both divisible by 9, but their difference $A - B$ is NOT divisible by 9.
4. Xavi says: I know a number X that is divisible by 4 and 3, but is NOT divisible by $4 \times 3 = 12$.
5. Yael says: I know a number Y that is is divisible by 4 and 6, but is NOT divisible by $4 \times 6 = 24$.
6. Albus says: I know a number A that is NOT divisible by 3, but twice that number IS divisible by 3.
7. Bert says, I know a number B that is even, but the number $3 \times B$ is NOT divisible by 6.
8. Cassie says, I know a number C , and $5 \times C$ is divisible by 3, but C is NOT divisible by 3.
9. Dwayne says, I know a number D and $15 \times D$ is divisible by 6. But D is NOT divisible by 6.

Party bags and trees

- 1) Josie has 30 cookies and 45 pretzels to put in party favor bags at her birthday party. If she needs to put the same number of cookies in each bag and the same number of pretzels in each bag, what is the largest number of bags she can use (so that she can invite as many friends as possible)?



- 2) Abe is going to plant 63 apple trees and 27 peach trees in rows that all have the same number of trees and are made up of only one type of tree. What is the greatest number of trees that Abe can have in each row?



Prime Decomposition

1) The number 1546 can be written as

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

or more efficiently, as $2^9 \times 3$, where the 9 means that 2 is multiplied by itself 9 times.

Is $2^9 \times 3$

- a. Divisible by 2?
- b. Divisible by 5?
- c. Divisible by 8?
- d. Divisible by 9?
- e. Divisible by 6?

2) A number can be written as $2 \times 3^2 \times 7^3 \times 13$. Is this number

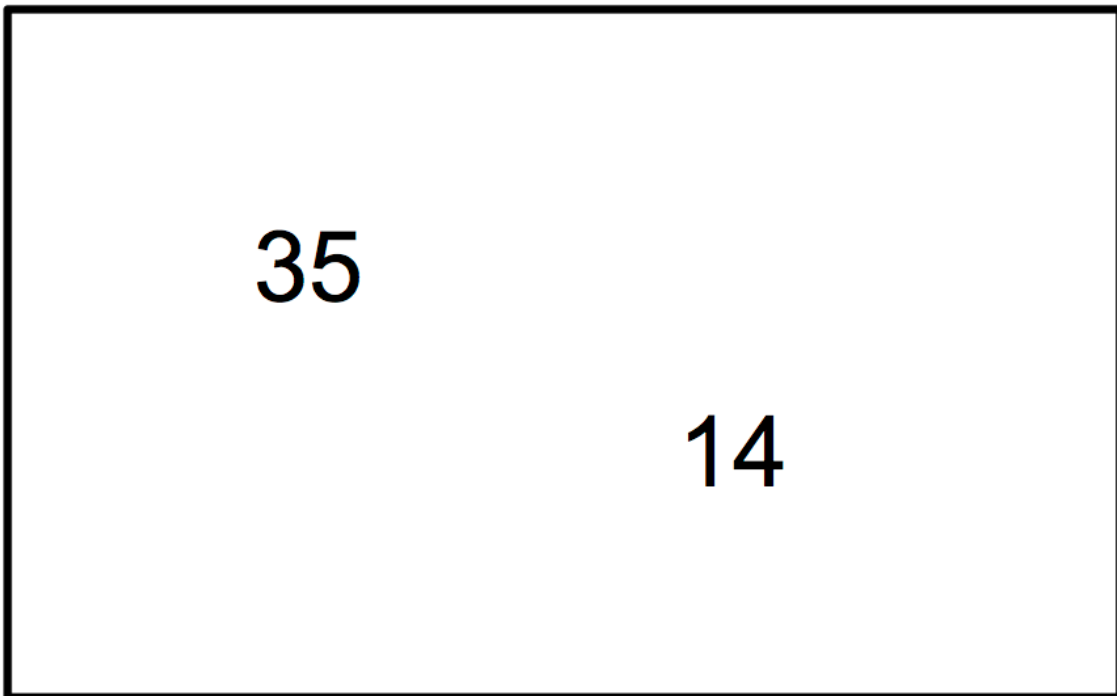
- a. Divisible by 2?
- b. Divisible by 4?
- c. Divisible by 14?
- d. Divisible by 12?

3) Two numbers are written as $2^2 \times 3 \times 7^3 \times 13$ and $2 \times 3^2 \times 7^2$.

- a. Is the first number divisible by the second?
- b. Is the product of these numbers divisible by 8? By 36? By 27? By 16? By 56?

Euclid's Game

Start with two numbers in a box. Two players take turns writing a new number in the box that is the difference of two existing numbers in the box and is a positive number (not zero or a negative number). The player that can no longer make a move loses.



For each pair of starting numbers below, decide if you want to go first or second.

- 8 and 3
- 10 and 6
- 30 and 45
- 63 and 27

Given two starting numbers, how can you decide which other numbers will appear in the box? How can you decide if the first or second player will win?