

Combinatorics

Day 1: Warm-up

You walk into the ice cream store. There are three flavors of ice cream: mint chocolate chip, cookie dough, rocky road. There are two kinds of cones: sugar and waffle. How many different types of ice cream cones can you order, if you are only allowed to order one scoop?



What if you are allowed to order two scoops, which can be different flavors or the same flavor?

1 Counting Fashion

1. You decide to wear a dress to school today. You own:

- 4 dresses, yellow, teal, pink, and violet (Y, T, P, V).
- 2 pairs of shoes: crocs and ugs (C, U).
- 3 pairs of socks, stripes, dots, and argyle (S, D, A).

How many different outfits can you choose between, if an outfit has to be one dress, one pair of shoes, and one pair of socks?



2. You also own

- 5 shirts: blue, white, red, orange, green (B, W, R, O, G),
- 3 pairs of pants: jeans, hiking pants, and leggings (J, H, L)

How many outfits can you choose between tomorrow, if tomorrow's outfit will be one shirt, one pair of pants, one pair of shoes and one pair of socks?

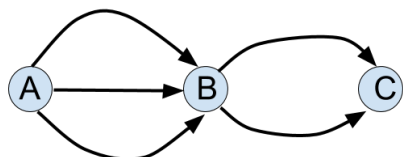
3. How many outfits total are possible?

Extra problems:

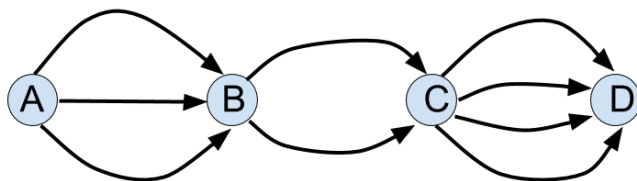
4. What if you can choose to wear socks or not?
5. What if you can also choose to wear pants under your dress or not?

2 Counting Road Trips

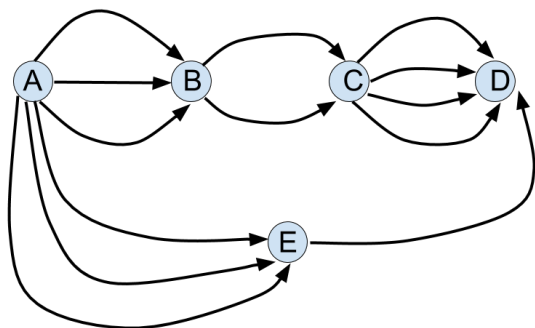
1. There are three towns A, B, C, on Frog Island. There are 3 roads from A to B and 2 roads from B to C. How many ways can you drive from A to C (without going back through A!)



2. A new town D is built on Frog Island, and 4 roads are opened leading from C to D. How many different ways can you drive from A to D? (Assume that you are only allowed to travel in the direction of the arrows.)

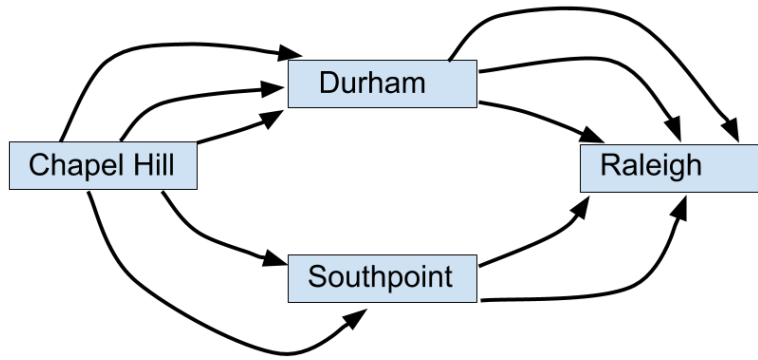


3. A new town E is built and 6 new roads as shown. How many different ways can you drive from A to D now?

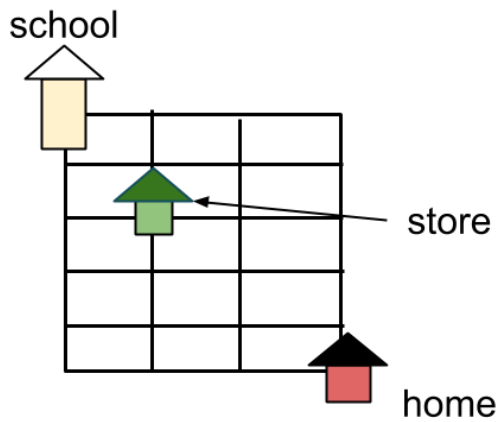


Extra Problems

4. How many ways are there to travel from Chapel Hill to Raleigh if only the roads drawn are open?



5. How many ways are there to walk from school to home, walking along grid lines, always heading south or east, if you have to stop by the store on the way?

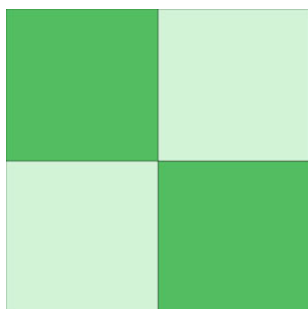


3 Counting Patterns

1. We toss a coin three times. How many different sequences of heads and tails can we obtain?



2. Each box in a 2×2 table can be colored black or red. How many different colorings of the table are there?



3. Bella has a big bag of black marbles and a big bag of white marbles. How many different black and white patterns can she make by placing four of these marbles in a row? (A pattern is a sequence of colors, for example *WWWW*, *BWBW*, and *WBBW* would be different patterns.)



4. What if there are three colors of marbles: black, white, and red? Then how many patterns can we make with four marbles in a row?

Extra Problems

5. A school has 677 students. Explain why at least 2 students must have the same pair of initials.
6. How many 3 digit numbers are there that have all odd numbers as digits? For example, 337 counts, but 352 does not.

Many of these problems are from *Mathematical Circles (Russian Experience)*, *A Decade of the Berkeley Math Circle - Volume 1*, and *Mathematicsl Circles Diary*