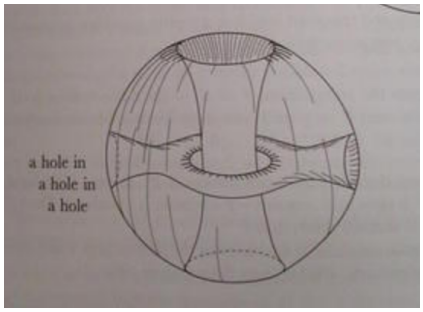
Topology of Surfaces - Part 2

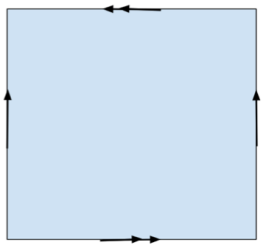
1 Warm Up Problems

1. What is the surface in this picture from Michael Spivak's Differential Geometry book?

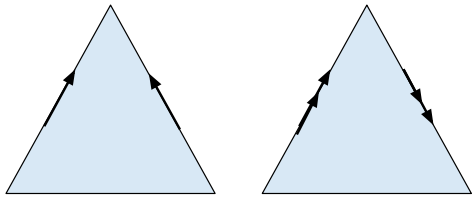


`http://www.kleinbottle.com/gallery/Spivak_Hole_Pix/IMG_3004`

2. What do you get when you cut a Klein bottle in half? Hint: it depends on how you cut it.

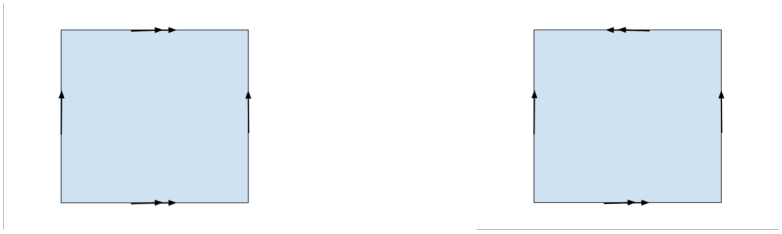


3. What two surfaces are obtained by gluing the edges of each triangle as shown? (The bottom edge doesn't glue to anything.)

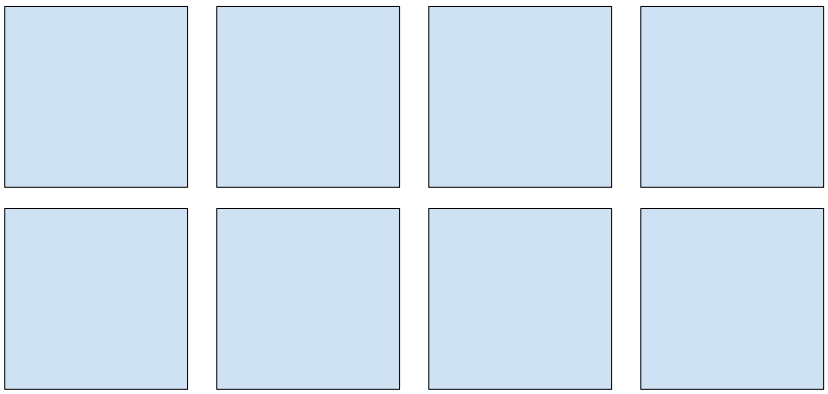


2 More Gluing Diagrams

Last time we drew arrows on a square to represent a torus and a Klein bottle.

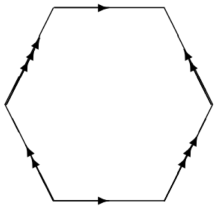


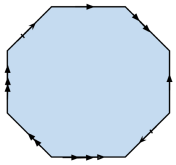
4. How many other ways are their to glue the edges of a square in pairs? Draw the arrows below. Only include essentially different ways, that might possibly give you a different topological surface.



5. Which of the gluing diagrams that you drew above represent non-orientable surfaces?
6. For each gluing diagram that you drew, count up the number of faces, edges, and vertices AFTER gluing.
   - When two edges are glued together, that only counts as one edge after gluing.
   - If several vertices end up glued together, that only counts as one vertex after gluing.
7. Which gluing diagrams have the same Euler characteristic and which have different ones?
8. Do the gluing diagrams with the same Euler characteristic actually represent the same surface? Prove your answer!

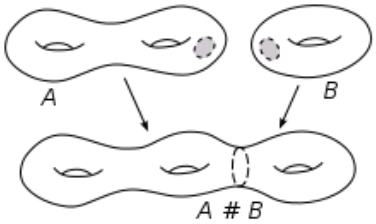
9. What surfaces do the gluing diagrams below represent?





10. What is the Euler characteristic of a 2-holed torus? Can you draw a gluing diagram to represent one?

11. To build the connected sum of two surfaces A and B, we cut out a disk from each surface and glue together the resulting boundary circles.



How can you compute the Euler characteristic of the connected sum $\chi(A\#B)$ from $\chi(A)$ and $\chi(B)$?

12. What do you get when you glue a disk (i.e. the inside of a circle) to the boundary circle of a Mobius band?

13. What do you get when you glue two Mobius bands together along their boundaries?

14. (a) Which of these gluing diagrams represent orientable surfaces and which represent non-orientable surfaces?

    (b) Which gluing diagrams represent the same topological surface?

    (c) Name the surfaces represented.