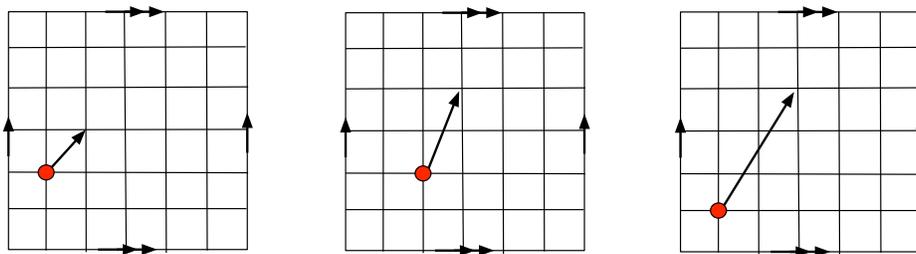


Topology of Surfaces

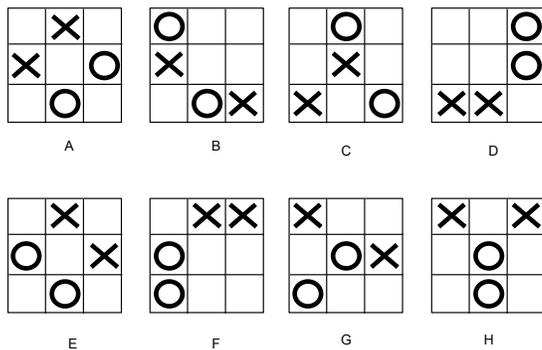
Most of this material is from *The Shape of Space* by Jeff Weeks. The word searches were taken from Torus Games at <http://geometrygames.org/TorusGames/index.html>. You can download the Torus Games app on your phone for many other games.

1 The Torus

1. A ladybug on a torus walks in a straight line until she returns to her starting point. Draw her path and find the length of her path if:
 - (a) she walks 1 unit northward for every 1 unit eastward, as shown at the left.
 - (b) she walks 2 units northward for every 1 unit eastward as shown in the center.
 - (c) she walks 3 units northward for every 2 units eastward as shown at right.



2. If the ladybug walks y units northward for every x units eastward, where y and x are integers, how long will her path be? What if y and x are not integers?
3. Which of the following positions are equivalent in torus tic-tac-toe?



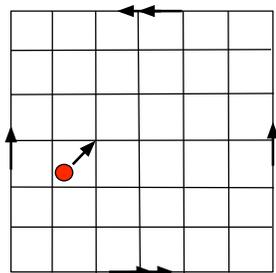
4. How many essentially different first moves are there in torus tic-tac-toe?
5. Is there a winning strategy for the first player in Tic Tac Toe on the torus? That is, is it possible for the first player to win no matter what the second player does?
6. A Cat's Game in Tic Tac Toe is a game where neither side wins, even though the board is filled up with X's and O's. Is it possible to have a Cat's Game in Tic Tac Toe on the torus?
7. Try this word search on the torus. Remember that the top and the bottom are glued together and the left and right sides are glued together.

h	l	m	e	a	i
n	a	l	n	b	r
j	l	d	a	e	a
c	a	t	w	m	t
x	e	g	i	p	a
o	s	s	u	m	p

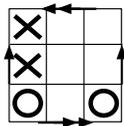
bat	pig
camel	possum
cat	puma
jaguar	rat
llama	seal
ox	shrew
panda	tapir

2 The Klein Bottle

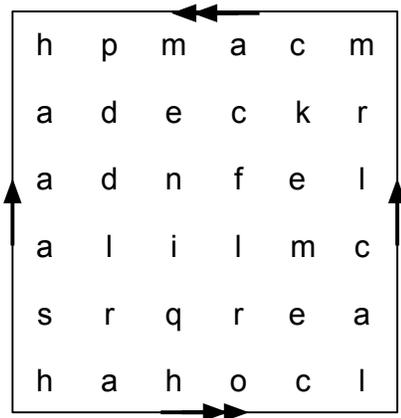
8. A ladybug on a Klein bottle walks in a straight line until she returns to her starting point. She walks 1 unit northward for every 1 unit eastward. Draw her path.



9. Where can X go to win immediately in Klein bottle Tic Tac Toe?



10. Play Tic Tac Toe on the Klein Bottle with a friend. Is it possible to get a Cat's Game? Is there a winning strategy for the first player? How many essentially different first moves are there?
11. Try this word search on the Klein bottle. Remember that the top and the bottom are glued together and the left and right sides are glued together, as the arrows show. (Hint: on a Klein bottle, letters and their mirror images are not distinguishable, so "d" and "b" are the same, and so are "p" and "q".)



ash	lilac
birch	maple
cedar	oak
elm	palm
fir	pine
larch	poplar

12. What do you get when you cut a Klein bottle in half? Hint: it depends on how you cut it!

3 The 3-Torus

13. Suppose a 3-torus is made from a rectangular block 600 km by 800 km by 2400 km.
- What is the shortest distance you can travel (in a straight line) to return to your starting place?
 - Suppose you want to construct two space stations as far away from each other as possible. Where should you put them? How far apart will they be?

4 Klein Space

14. Find a way to embed a large 2-dimensional Klein bottle inside Klein space without self-intersections. Does it have one side or two sides?