



Chapel Hill Math Circle

Intermediate Group

In the Land of Logic¹, Part 1

September 10, 2016

Let us visit the logical land of knights and knaves, where knights always tell the truth and knaves always lie. Moreover, each inhabitant is either a knight or a knave.

Problem 1 (A Classic Case). On the day of his arrival, Abercrombie came across three inhabitants, whom we will call A, B and C. He asked A: "Are you a knight or a knave?" A answered, but so indistinctly that Abercrombie could not understand what he said. He then asked B: "What did he say?" B replied: "He said that he is a knave." At this point, C piped up and said: "Don't believe that; it's a lie!" Was C a knight or a knave?

Problem 2 (A Variant). According to another version of the story, Abercrombie didn't ask A whether he was a knight or a knave (because he would have known in advance what answer he would get), but instead asked A how many of the three were knaves. Again A answered indistinctly, so Abercrombie asked B what A had said. B then said that A had said that exactly two of them were knaves. Then, as before, C claimed that B was lying. Is it now possible to determine whether C is a knight or a knave?

Problem 3. Next, Abercrombie met just two inhabitants, A and B. A made the following statement: "Both of us are knaves." What is A and what is B? Remark. "But," some of you might say (after having read the solution to Problem 1.1), "how is this situation possible? You have already proved that no inhabitant of the island can claim to be a knave, so how can an inhabitant claim that both are knaves, when he can't even claim that he is a knave?" This is an important point that is fully discussed in the solution.

Problem 4. According to another version of the story, A didn't say "Both of us are knaves." All he said was "At least one of us is a knave." If this version is correct, what are A and B?

¹ These problems are taken from Raymond Smullyan's *Logical Labyrinths*. If you're interested in logic from a philosophical or mathematical point of view, I highly encourage you to read anything and everything from him. Smullyan is amazing!

Problem 5. According to still another version, what A actually said was "We are of the same type—that is, we are either both knights or both knaves." If this version is correct, then what can be deduced about A and B?

Problem 6. On one occasion, Abercrombie came across two natives who were lazily lying in the sun. He asked one of them whether the other one was a knight and got an answer (yes or no). He then asked the other native whether the first one was a knight, and got an answer (yes or no). Were the two answers necessarily the same?

Problem 7. On another occasion, Abercrombie came across just one native who was lazily lying in the sun. Abercrombie asked the native his name, and the native replied: "John." Was the native a knight or a knave?

Problem 8. On another occasion, Abercrombie came across a native and remembered that his name was either Paul or Saul, but couldn't remember which. He asked him his name, and the native replied "Saul." From this, it is not possible to tell whether the native was a knight or a knave, but one can tell with very high probability! How? (This is a genuine problem, not a monkey trick!)

Problem 9. In the next incident, Abercrombie came across three natives, A, B, and C, who made the following statements: A: Exactly one of us is a knave. B: Exactly two of us are knaves. C: All of us are knaves. What type is each?

Problem 10. (Who Is the Chief?). Abercrombie knew that the island had a chief and was curious to find him. He finally narrowed his search down to two brothers named Og and Bog, and knew that one of the two was the chief, but didn't know which one until they made the following statements: Og: Bog is the chief and he is a knave! Bog: Og is not the chief, but he is a knight. Which one is the chief?