Chapel Hill Math Circle

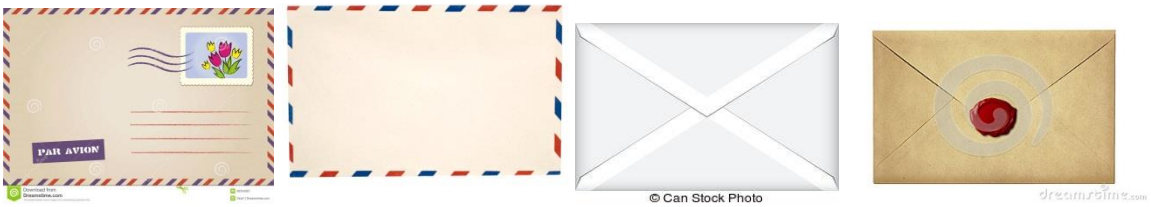
THINK MATH STUDIO

Intermediate Group

April 15, 2017

1. Statement: **Every envelope that has a stamp in the front, also has a seal on the back.**
   You have these four envelopes in front of you: two are face up, one has a stamp and the other does not; two are face down, one has a seal and the other does not.

   #1 #2 #3 #4



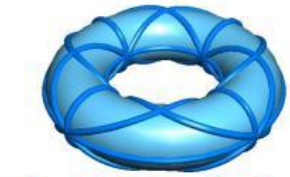
   As a group, make a decision: Which envelopes do you need to turn to verify that the statement above is correct?

2. Among the following numbers, choose a number which is different from the others. Explain why it is different. As a group, try to find as many possible cases or answers as you can.

   2 3 4 6 8 12

3. Kayla needs to cook 3 pancakes on both sides. Each side takes 1 min to cook, but only two pancakes can fit in the pan. What is the fastest time she can do this?

4. There are two cups on the table, one with 200 ml of coffee, and one with 200 ml of milk. Wade takes 1 tablespoon (15 ml) of milk from the cup and pours it into the coffee. He stirs it well with the coffee, then takes 1 table spoon of coffee with milk and pours it back into milk, and stirs it well. He repeats this process one more time, ending up with the same amount of liquid in both cups (200 ml). In the end, is there more milk in the coffee or more coffee in the milk?



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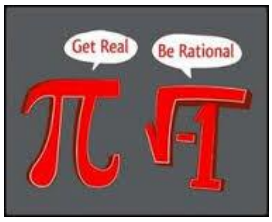

5. Nora and Tim started walking towards each other at the same time from the two opposite ends of a path in the park. At the same moment, a dog named Aisha starts running from Nora towards Tim, and as she reaches Tim, she runs back to Nora, and so forth, until they all meet. The distance between Nora and Tim in the beginning is 18 km. Nora walks with the speed of 4 km per hour and Tim walks with the speed of 5 km per hour. The dog runs with a speed of 6 km per hour. What distance will the dog have run by the time they all meet?

6. A man is running through a train tunnel. When he is 2/5 of the way through, he hears a train that is approaching a tunnel from behind him at a speed of 60 mph. Whether he runs ahead or runs back, he will reach an end of the tunnel at the same time the train reaches that end. At what speed, in mph, is he running? (Assume he runs at a constant speed.)

7. There are five integers *a*, *b*, *c*, *d*, and *e*. The sums of all possible pairs of these integers are

   0, 2, 4, 4, 6, 8, 9, 11, 13, 15.

   a) What are these integers?

   b) Can you find five integers whose sums of all possible pairs are

      12, 13, 14, 15, 16, 16, 17, 17, 18, 20?

8. Find the sums.

   $2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^n$

   $3^0 + 3^1 + 3^2 + 3^3 + \ldots + 3^n$

   $k^0 + k^1 + k^2 + k^3 + \ldots + k^n$