The Triangle Inequality[1]
January 28, 2017

Warmup problems

1. You have sticks of length 1cm, 2cm, 3cm, and 4cm. If you are allowed to reuse them, how many different triangles can you make using the sticks as sides? List them.
2. What if the lengths of the sticks are 1cm, 2cm, 2cm, 2cm, and 3cm?
3. A hinge connects two rods (of lengths 3 and 5) that rotate around the ball joint. What are the minimal and maximal distances between the endpoints of the rods?

Problems

4. ★ Based on the warmup problems, what conjecture(s) can you make? Prove or disprove them.
5. Prove that any side of a triangle is shorter than half of the perimeter of that triangle.
6. You have sticks of length 1cm, 2cm, 3cm, 4cm, and 5cm. If you are allowed to reuse them, how many different triangles can you make using the sticks as sides? List them.
7. Points A, B, C, and D are given on a plane. Prove that AD ≤ AB + BC + CD.
8. Four houses A, B, C, and D are the vertices of convex quadrilateral ABCD. Where should a well X be built so that the sum of the distances from every house to the well is the smallest possible?
9. Prove that in a quadrilateral, the sum of the lengths of the diagonals is smaller than the perimeter of the quadrilateral.
10. Is it possible for a straight line to cross all four sides of a nonconvex quadrilateral? Explain your reasoning and convince your peers.
11. ★ Is it possible for a straight line to cross all five sides of a nonconvex pentagon? Explain your reasoning and convince your peers.

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[1]Problems 1, 2, 4, and 6 are our own. The other ones are based on Alexander Shen's *Geometry in Problems*, from MSRI's Mathematical Circles Library.