

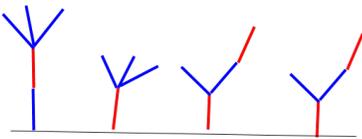
Hackenbush, Toads and Frogs, and Cutcake*

1 Warm-Up

In Red-Blue Hackenbush, there are two players named Red and Blue who alternate moves. Red moves by cutting a red segment and Blue, by cutting a blue one. The cut segment is deleted together with any other segments that are no longer connected to the ground.

A player loses when they can't make a move (because there are no longer any segments of their color).

Is the game below better for blue, better for red, or an even game? What is the best first move if blue moves first?



Try to analyze the game by assigning numbers to each stalk like we did last week.

2 Infinite Hackenbush

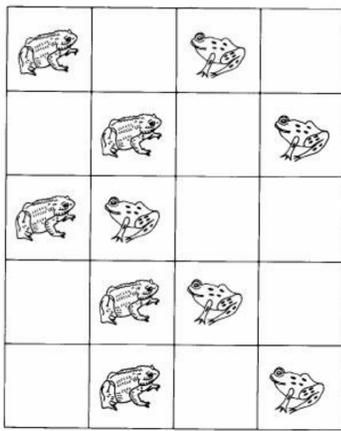
- Suppose there are two infinitely-long stalks, one all of red segments and the other all blue.
 - Is this game better for Red, better for Blue, or equal?
 - What is the best strategy?
- Suppose there are two infinitely-long stalks, one all of red segments and the other all blue, together with one single red segment. If Red goes first, what is Red's best move?
- What number should be assigned to an infinitely long stalk with one blue segment at the bottom and the rest red? Hint: consider finite stalks that contain a single blue segment at the bottom, and a longer and longer sequence of red segments above it, How many of these stalks would be needed to perfectly balance out a single red segment to make a 0 game?
- What number should be assigned to an infinitely-long stalk of alternating blue and red segments, starting with blue? Build a balanced 0 game with some of these stalks and some single segments, and try playing it with your neighbor.

*All of these games are from *Winning Ways for your Mathematical Plays, Volume 1* by Berlekamp, Conway, and Guy

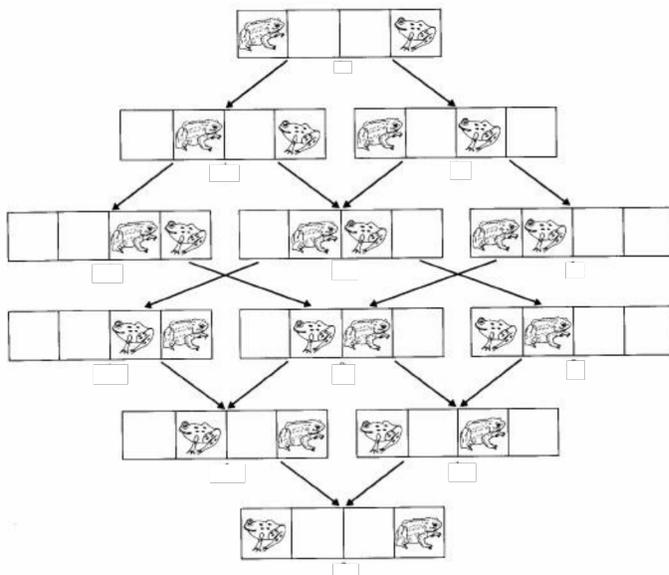
3 Toads and Frogs

In Toads and Frogs, one player moves toads and one player moves frogs. Toads can only move from west to east and frogs can only move from east to west. Each creature can either move to the next square, if it is open, or can jump over one opposing creature onto an empty square. Once a frog or toad is off the game board, it can no longer move. The player unable to move loses.

5. What is the best move in the following game if frogs moves first? If toads moves first? Is the game better for frogs or toads?



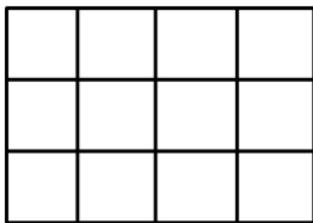
6. Find a way to assign numbers to positions as in the other games. What number gets assigned to the game above? You may want to fill in numbers for each position in the chart below to help you.



4 CutCake

In CutCake, Vertie and Horrie take turns breaking a rectangle along horizontal and vertical cut-lines. Vertie can break any rectangle along a vertical line, and Horrie can break any rectangle along a horizontal line. (Once there are several rectangles in the game, a player can choose any of the rectangles to cut.) The player who can no longer make a move loses.

7. Try playing a game with a 3x4 rectangle. Do you think the game better for Vertie or Horrie or neither?



8. Try assigning numbers to games like we did for Hackenbush. Assign positive numbers to games that are better for Vertie and negative numbers to games that are better for Horrie, and 0 to games that are fair (e.g. the second player always loses).

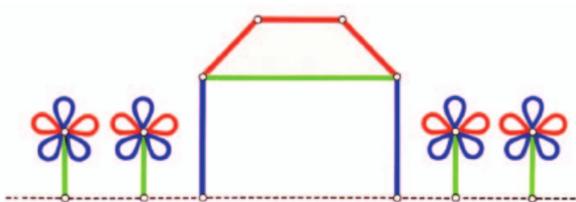
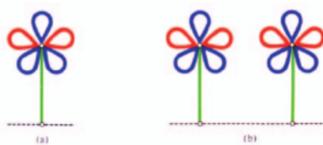
Fill out this chart, with the numbers. The row number is the height and the column number is the width of the rectangle.

	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
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12												

9. I challenge you to a game of 5 x 4 cutcake. I'll let you choose if you want to be Vertie or Horrie, and I get to choose whether to go first or second. Do you want to play me? If so, do you want to be Vertie or Horrie?

5 Red Blue Green Hackenbush

In Red Blue Green Hackenbush, green segments can be taken by either player. Try playing the one flower game, the two flower game, and the flower garden and house games. What numbers would you assign to these games? How is Red Blue Green Hackenbush different from Red Blue Hackenbush?



6 Your Turn!

- Invent your own two player game and analyze it.