# Battle of Xorks and Yorks[1]
April 8, 2017

## Warmup Problem

1. A certain type of bacteria multiplies so fast that the area they cover in a Petri dish doubles every 10 minutes. One bacterium was placed in a Petri dish and the dish became completely covered with bacteria in exactly 5 hours. How long will it take for the dish to get completely covered if we start with two bacteria?

## Problems

2. Fifteen black dots have been marked on a square of white paper of side 4 cm. Prove that it is possible to cut out a square of side 1 cm from the larger square so that the smaller square will not contain any dots inside.

3. A flock of white geese flew over a string of lakes. As they came to each lake, half of the remaining geese plus a goose landed on that lake, while the others flew on. By the seventh lake, all the geese had landed. How many geese were in the flock?

4. Draw 4 dots so that any 3 dots will be vertices of an isosceles triangle. Can you do it with 5 and 6 dots?

5. A 3 x 3 x 3 cube is to be cut into 27 unit cubes. What is the least number of cuts needed to do this? (You are allowed to rearrange the pieces after each cut. Assume each cut is parallel to a face of the cube.

6. On a stormy night, ten guests came to a dinner party and left their shoes in the foyer to keep the carpet clean. After dinner, there was a blackout and the guests, leaving one by one, randomly put on any pair of shoes big enough for their feet. (Each pair of shoes stays together.) Any guest who couldn't find a pair of shoes that fit them, spent the night. What is the largest number of guests who might have had to spend the night?

7. A Xork lands on a planet that contains 100 Yorks. Every day after that, a battle takes place where each Xork destroys one York; right after the battle, every Xork and every York split into two. Prove that, sooner or later, all Yorks will be destroyed. How long will it take for that to happen?

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[1] Based on Sergey Dorichenko's *A Moscow Math Circle*, from MSRI's Mathematical Circles Library.