Chapel Hill Math Circle

Intermediate Group

More Reasoning and Proofs[1]

March 25, 2017

Warmup problem

1. A train that is 180 meters long passes a signal in 90 seconds. How long will it take it to cross a bridge that is 360 meters long?

Problems

2. Four posts have been placed at the corners of a square pond. How can the pond be expanded without removing the posts so that the area doubles, the shape remains a square, and the posts don't end up in the water?

3. There are 25 students in a class.
   a. Prove that two students have birthdays in the same month.
   b. Are there necessarily three such students?

4. Find the remainder when:
   a. 3^{100} is divided by 5
   b. 5^{100} is divided by 3

5. Suppose that 26 numbers are chosen from the set 1, 2, …, 50. Must two of the numbers differ by 1?

6. How many different 10-digit numbers can be written using only the digits 1 and 2?

7. There are 10 mangoes in a tree. How many ways are there to pick several of them?

8. A 200 x 3 rectangle is drawn on graph paper along the grid lines. How many grid squares will a diagonal of the rectangle cross?

9. In a class with at least two students, are there at least two students with an equal number of friends in the class? Prove it!

[1] Based on Sergey Dorichenko's *A Moscow Math Circle*, from MSRI's Mathematical Circles Library.