



Chapel Hill Math Circle

*Intermediate Group*

## Reasoning with Proofs<sup>1</sup>

March 18, 2017

### Warmup problem

1. One of the diagonals of a rhombus is equal to a side. What are the measures of the angles of the rhombus?

### Problems

2. Can the square of an integer end with a 2? Explain your answer.
3. "There are 25 students in our class, and each is friends with exactly seven classmates." Why can't that statement be true?
4. Prove that if you have a collection of 11 natural numbers, there must be two of those numbers whose difference is divisible by 10.
5. From the set of numbers 1, 2, 3, ..., 50, twenty-six numbers are chosen. Prove that one of the numbers is divisible by another.
6. Carol is on an airplane flight. First she read a book; then she slept; then she looked out of the window; and then she drank orange juice. Each of these activities, except the first book, took exactly half as long as the previous one. She started reading her book at noon and finished her orange juice at 1:00 pm. When did Carol start looking out of the window?
7. More than half of the surface of the spherical planet Urth is land. Assuming Urth is solid beneath its surface, prove it is possible to dig a straight tunnel through the center of Urth and connect land with land.

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<sup>1</sup> Based on Sergey Dorichenko's *A Moscow Math Circle*, from MSRI's Mathematical Circles Library.