



Chapel Hill Math Circle

## Intermediate Group

### Counting - Second Part<sup>1</sup> March 11, 2017

#### Warmup problem

1. Can you cut 27 inches from a 144-in ribbon without a ruler? How?

#### Problems

2. There are 25 students in a class. In how many different ways is it possible to choose the following groups from the class?
  - a. Two hall monitors.
  - b. Three hall monitors.
  - c. A hall monitor and the president.
3. A company has 67 employees. Of them, 47 speak Spanish, 35 speak German, and 23 speak both.
  - a. How many employees don't speak either language?
  - b. Suppose that, in addition, 20 employees speak French, 12 speak French and Spanish, 11 speak French and German, and 5 speak all three. How many employees don't speak any of the three languages?
4. You want to cover two concrete staircases, both one meter high and two meters long. The first staircase has 7 steps and the second one has 9. Is it possible to cover the second staircase with the same amount of carpet than the first one?
5. Is it possible to connect 7 light bulbs to a power source using only 3 switches so that it would be possible to light up any number of bulbs, from 0 to 7?
  - a. What if there are 8 bulbs and 3 switches?
6. A toy factory produces multicolored triangular pyramids (what's another name for those pyramids?). Each pyramid has four equilateral triangular faces with one yellow, one red, one blue, and one green.
  - a. How many different coloring patterns can the factory produce on these triangular pyramids?
  - b. What if the factory produces cubes having square faces, each of a different color?
  - c. In how many ways can you arrange the numbers 1-6 on a regular die?
7. ★ A group of 15 children gathered 100 nuts. Prove that 2 of them have gathered an equal number of nuts.

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<sup>1</sup> Based on Sergey Dorichenko's *A Moscow Math Circle*, from MSRI's Mathematical Circles Library.