



Chapel Hill Math Circle

Intermediate Group

In the Land of Charts¹: Truth Tables

October 8, 2016

A census taker, Mr. McGregor, once did fieldwork on the Island of Knights and Knaves. On this island, women are also called knights and knaves. McGregor decided on this visit to interview married couples only.

1. (And) McGregor knocked on one door; the husband partly opened it and asked McGregor his business. "I am a census taker," replied McGregor, "and I need information about you and your wife. Which, if either, is a knight, and which, if either, is a knave?"
"We are both knaves!" said the husband angrily as he slammed the door.
What type is the husband and what type is the wife?
2. (Or) At the next house, McGregor asked the husband: "Are both of you knaves?" The husband replied: "At least one of us is." What type is each?
3. (If-Then) The next home visited by McGregor proved more of a puzzler. The door was opened timidly by a rather shy man. After McGregor asked him to say something about himself and his wife, all the husband said was: "If I am a knight, then so is my wife."
McGregor walked away none too pleased. "How can I tell anything about either of them from such a noncommittal response?" he thought. He was about to write down "Husband and wife both unknown," when he suddenly recalled an old logic lesson from his Oxford undergraduate days. "Of course," he realized, "I can tell both their types!"
What type is the husband and what type is the wife?

¹ Problems taken from Raymond Smullyan's *Forever Undecided*. This book is a logic puzzle tour through the enchanted land exposed by Gödel's Second Incompleteness Theorem.

4. (If-and-Only-If) When the census taker interviewed the fourth couple, the husband said: "My wife and I are of the same type; we are either both knights or both knaves." (The husband could have alternatively said: "I am a knight if and only if my wife is a knight.") What can be deduced about the husband and what can be deduced about the wife?

5. What statement could a native make from which you could deduce that is he is a knight, then there is gold on the island, but if he is a knave, then there might or might not be gold on the island? (Assume that every native knows whether there is gold or not on the island.)

6. What statement could a native make such that you could deduce that if there is gold on the island, then he must be a knight, but if there is no gold on the island, then he could be either a knight or a knave?

7. I once visited this island and asked a native: "Is there gold on this island?" All he said in reply was: "I have never claimed that there is gold on this island." Later, I found out that there was gold on the island. Was the native a knight or a knave?