Chess Problems  
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For: Chapel Hill Math Circle Grades 6-7

Chess Piece Movement:

The pawn is one of the trickiest pieces in chess. It only moves forward once space at a time, however it can only capture the opponent’s piece if it 1 space away on either forward diagonal. If there is a piece blocking the pawn’s forward movement, it can’t move through unless the blocking piece moves or it can capture along one of its forward diagonals. There are a few more rules to a pawn’s movement, however you should look them up on your own time, they can get tricky.
Independence: Maximum number of pieces that can be placed on a board without attacking each other (when all pieces are of the same type).

Find the independence board for...
   a. Pawn
   b. King
   c. Bishop
   d. Rook
   e. Knight
   f. Queen

Dominance: Minimum number of pieces to cover board where all squares are attacked by at least one piece (when all pieces are of the same type).

Find the dominance board for...
   a. King
   b. Rook
   c. Bishop
   d. Knight
   e. Queen
   f. Pawn

Tour: A path a piece can take to go around to each square on the board without going on the same square twice.

Find the tour for...
   a. King
   b. Bishop
   c. Rook
   d. Queen
   e. Pawn
   f. Knight

Is it possible to return to the square you began at? Verify for each piece. Why or why not?

If you vary the dimensions of the board to 7x7, 9x9, 10x10, 4x6, 3x10, and so on, how does that affect the ability of a piece to take a tour? Please focus on the knight tour. Does it matter where you start? And are there dimensions where it is impossible? What if you remove 2 corners? 4 corners?