

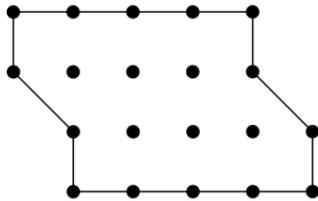
# Math Auction

## 1 Rules of the Math Auction

1. We divide into teams and work for a fixed amount of time to solve the problems below.
2. Each team is given \$200 to start.
3. The best solution to a problem is worth \$100.
4. The problems are put up for auction in the order given. The team with the highest bid is allowed to present its solution.
5. The problem is then put up for bid again (and again), but each time the solution must be better than the previous solution.
6. When no other team wants to buy the problem, the team with the best solution collects the value of the problem. Every team that bought the problem pays for its bid, even if it did not have the winning solution.
7. If a team can prove that it has found the best solution (by showing that no better solution is possible), then that team gets an additional \$25 prize money for the problem.

## 2 Problems

1. Find as many different ways as possible to cut this figure into four pieces of the same size and shape, using straight line segments that begin and end at grid points.



*A team has a stronger solution for this problem if it is able to demonstrate additional ways to cut the figure.*

2. Suppose we have a balance (i.e. a scale with two arms, which can only tell us whether the items place on one arm weigh less than or more than the items place on the other arm). We are given 16 coins, one of which is counterfeit and weighs more than its real counterparts. Present an algorithm to find the counterfeit coin in the fewest number of weighings as possible in the worst case.



*A team has a stronger solution for this problem if it has a method that requires fewer weighings, even in the worst possible case.*

3. Six children sit around a circular table with 6 seats. Find as many different ways as possible to rearrange the children so that no child moves more than one seat to the right or to the left of his or her original position.

*A team has a stronger solution for this problem if it is able to demonstrate additional ways to rearrange the children.*

4. Make the minimum possible number of marks on a wooden plank so that every integer number of inches from 1 through 15 can be measured using this plank; that is, this length can be represented as the distance between some pair of the marks.

*A team has a stronger solution for this problem if it has a way using fewer marks.*

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