Warm-up Problems

1. Is it possible to put 5 points on a plane so that there are only two distinct distances between pairs of points?

2. At a large school, there are 1000 lockers, all on one wall of a long corridor. The lockers are numbered, in order, $1, 2, 3, \ldots, 1000$, and to start, each locker is closed. There are also 1000 students, also numbered $1, 2, 3, \ldots, 1000$. The students walk the length of the corridor, opening and closing lockers according to the following rules:
   Student 1 opens every locker
   Student 2 closes every second locker, that is, lockers $2, 4, 6, 8, \ldots$
   Student 3 changes the state of every third locker, closing it if it is open and opening it if it is closed
   ...
   ...
   ...
   Student n changes the state of every nth locker
   Etc.

   When all 1000 students have walked the corridor, which lockers end up open?

Four Points, Two Distances

Find all the different ways to arrange 4 points in the plane so that there are only two distinct distances between pairs of points.

(turn over)

Variations on the Locker Problem

1. (a) After all 1000 students go down the hall, which lockers are open?

   (b) If the students go down in a different order, is the result changed?

   (c) What if student 3 is sick and has to miss her turn? What if she takes a second turn while the teacher's not looking?

   (d) What if students 3 and 9 are ill? 3 and 10?

2. Suppose that we can send any students we like down the corridor. If, when we are done, we want only locker 1 open and all the others closed, then which students should go?

3. Which students should we send if we only want locker 3 open? Only locker 9? Both 3 and 9 and nothing else?

4. Suppose we want only the lockers with prime numbers open. Which students should be sent?

5. Let L be any subset of $\{1, 2, 3, \ldots, 1000\}$, the set of the first 1000 positive integers. Is there a set of students that you can send down the corridor so that when all of these students have gone, the set of open lockers is exactly those with a number in L?

6. Let S_1 and S_2 be two different groups of students, which may or may not have some students in common. Each is sent down a row of lockers. Is it possible that the students from the two groups leave exactly the same lockers open?

7. What if we want to send students so that only lockers with perfect cube numbers are open?

8. Suppose that we send down the corridor exactly those students with perfect square numbers (e.g. 1, 4, 9, 16, ...). Which lockers are left open when this activity is concluded? What if we send students whose numbers are perfect cubes?

9. What other interesting questions can you ask about the locker problem?

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