

## Part 1: Nim and Jim, Continued

Remember the games from last time:

1. **Three Pile Nim.** There are three piles of pennies. Two players take turns removing any number of pennies from any one of the three piles. The player unable to move loses.
2. **Three Row Jim.** A Jim game starts with three rows of pennies, some (or all) turned heads up, some (or all) tails up. Players alternate moves: select ONE row, and flip over one or more pennies, changing some heads to tails and/or some tails to heads. RULE: The first change from the left must be a head to a tail (but it does not need to be the leftmost head). The player unable to move loses.

We saw that Three Pile Nim and Three Row Jim are isomorphic games, where the isomorphism is given by writing the pile sizes in binary (base 2). For example, the Nim game with pile sizes 6, 5, and 3, is equivalent to the Jim game with rows:

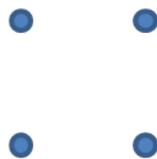
$$\begin{array}{ccc} 1 & 1 & 0 & & H & H & T \\ 1 & 0 & 1 & \text{OR} & H & T & H \\ 0 & 1 & 1 & & T & H & H \end{array}$$

since  $6_{10} = 110_2$ ,  $5_{10} = 101_2$ , and  $3_{10} = 11_2$ .

Write as many winning and losing positions of Nim and Jim on the board as possible, and try to identify patterns and a winning strategy.

## Part 2: Four Points, Two Distances

Consider four points in the plane, located at the corners of a square. How many distinct distances between pairs of points are there, in this arrangement?



Apart from the corners of a square, can you find other ways of arranging four points in the plane, such that there are only two distinct distances between them? How many different arrangements can you find?

Thanks to Alon Amit for the Four Points Two Distances Problem.

See <https://affineness.wordpress.com/2009/01/27/four-points-two-distances/> .