

Warm up Problem. What is the minimum number of weights which enable us to weigh any integer number of grams of gold from 1 to 100 on a standard balance with two pans? Weights can only be placed on the left pan.

Nim and Jim

1. **Puppies and Kittens.** There are two piles of pennies; one pile contains 10 and one contains 7. A player can take any number of pennies from the first pile (the puppies), or any number from the second pile (the kittens), or the player can take the same number of pennies from both piles. For example, a player could take 2 from the first pile, or 6 from the second pile, or 3 from each pile. The player unable to move loses. Generalize to other starting positions.
2. **Three Pile Nim.** There are three piles of pennies; one pile with 6 pennies, a second pile with 5 pennies, and a third pile with 3 pennies. Two players take turns removing any number of pennies from any one of the three piles. The player unable to move loses. Find a strategy to win the game with this starting position, and generalize to other starting positions.
3. **Three Row Jim.** A Jim game starts with three rows of pennies, some (or all) turned heads up, some (or all) tails up. Players alternate moves: select ONE row, and flip over one or more pennies, changing some heads to tails and/or some tails to heads. RULE: The first change from the left must be a head to a tail (but it does not need to be the leftmost head). The player unable to move loses. Equivalently, if you see only tails, then you've lost! Note: both players sit at the same side of the board, so left means the same side for each player. Which starting positions allow the first player to win?

For example, if H = heads and T = tails:

H H T		H T T		
T H T	→	T H T		This is a legal move.
T T H		T T H		

H H T		H T H		
T H T	→	T H T		This is a legal move.
T T H		T T H		

H H T		H H T		
T H T	→	H H T		This is NOT a legal move.
T T H		T T H		

Jim was invented by Bard math professor Japheth Wood.